Tutor: Robert Ramspeger

Aufgabe 14.1

a) $_{29}Cu = 29 \cdot e = 4{,}65 \cdot 10^{-18}\,C$

b) $\rho_{Cu} = 8{,}92\ g/cm^3$ $\quad V = \frac{m}{\rho} = \frac{3g}{8{,}92\,g/cm^3} = 0{,}336\,cm^3$

$V_m = 7{,}11 \cdot 10^{-6}\,\frac{m^3}{mol}$ $\quad V_m = \frac{V}{n}$ $\quad n = 0{,}0472\,mol$

mit Avogadrozahl

$\rightarrow 2{,}845 \cdot 10^{22}$ Atome

$132008\,C = Q_{Cu}$

c) $P = U \cdot I$ $\quad 220V, 40W$

$I \rightarrow 0{,}18\,A$ $\quad I = \frac{dQ}{dt}$

$t = 203{,}71\,h \approx 8{,}4\,d$

Aufgabe 14.2

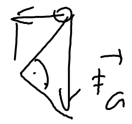
a) $F = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} = 2{,}31 \cdot 10^{-16}\,N$

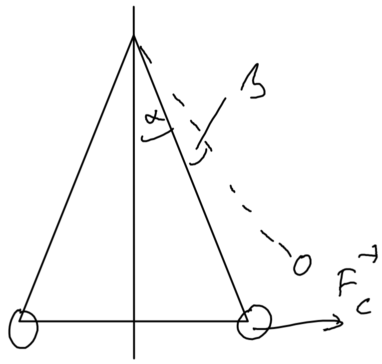
b) $\vec{F_g} = -\vec{F_c} = -\frac{G(m_1 m_2)}{r^2} \cdot \hat{r}$

$m_1 = m_2 = \sqrt{3{,}46 \cdot 10^{-18}\,kg} = 1{,}86 \cdot 10^{-9}\,kg$

Aufgabe 14.3

a)





$|\vec{F}_r| = \sin\alpha \cdot m \cdot g$

$|\vec{F}_{ges}| = \cos\alpha \cdot \sin\alpha \cdot m \cdot g \overset{!}{=} \frac{Q^2}{4\pi\varepsilon_0 d^2}$

$\rightarrow Q = \pm 2 d \sqrt{\pi \varepsilon_0 \cos\alpha \cdot \sin\alpha \cdot m \cdot g}$

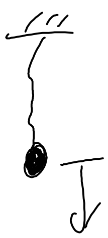
$= \pm 5{,}9 \cdot 10^{-5}\,C$

b) harmonische Schwingung

$F = -D \cdot x$

$m \cdot a = -D \cdot x$

$a = \dot{v} = \ddot{x}$



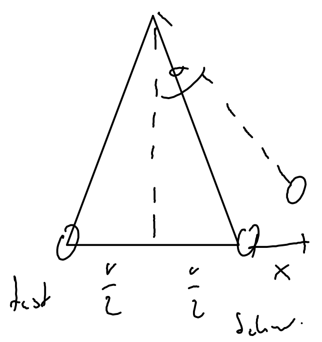
$\boxed{m\ddot{x} = -Dx}$ DGL $\rightarrow$ LSG $x(t) = A\cos(\omega t + \delta)$

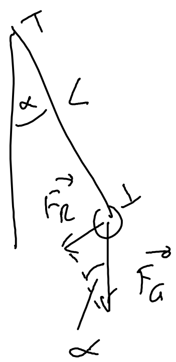
$\ddot{x}(t) = -A\cos(\omega t + \delta)\omega^2 = -\omega^2 x(t)$

In DGL:

$-m\omega^2 x = -Dx \Rightarrow m\omega^2 = D$

$\hookrightarrow \omega = \sqrt{\frac{D}{m}}$





$F_R = \sin\alpha \, F_G$

$= \sin\alpha \, mg$

$= \frac{x + \frac{r}{2}}{L} mg$

$F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(r+x)^2}$

$$F_{ges} = F_C - F_R = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(r+x)^2} - \frac{x+\frac{r}{2}}{L}mg$$

$$= \frac{1}{4\pi\varepsilon_0 r^2}\frac{q^2}{\left(1+\frac{x}{r}\right)^2} - \frac{mgr}{2L}\left(1+\frac{2x}{r}\right)$$

$$\frac{1}{4\pi\varepsilon_0 r^2}\frac{q^2}{\left(1+\frac{x}{r}\right)^2}$$

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2}f''(x_0)(x-x_0)^2 + \dots$$

$$f(x) \approx \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(x_0)(x-x_0)^n$$

$$\frac{1}{\left(1+\frac{x}{r}\right)^2}(0) \approx 1 - 2\frac{1}{r\left(1+\frac{0}{r}\right)^3}(x-0) = 1-2\frac{1}{1^3 r}\cdot x = 1-\frac{2x}{r}$$

$$F_C = \frac{q^2}{4\pi\varepsilon_0 r^2}\left(1-2\frac{x}{r}\right) \qquad F_G = \frac{mgr}{2L}\left(1+2\frac{x}{r}\right)$$

Aus a) Für $x=0$ gilt: $\boxed{\frac{q^2}{4\pi\varepsilon_0 r^2} = mg\frac{r}{2L}}$

$$F_{ges} = F_C - F_G = \frac{mgr}{2L}\left(\cancel{1}-2\frac{x}{r}\right) - \frac{mgr}{2L}\left(\cancel{1}+2\frac{x}{r}\right)$$

$$= -4\frac{x}{r}\frac{mgr}{2L} = -\frac{2mg}{L}x$$

$$\left.\begin{array}{l} F = -Dx \\ F = -\frac{2mg}{L}x \end{array}\right\} \quad D = \frac{2mg}{L}$$

$$\omega = \sqrt{\frac{D}{m}} = \sqrt{\frac{2g}{L}} = 4{,}43\,\frac{1}{s}$$

$$f = \frac{1}{2\pi}\omega = 0{,}7\,\frac{1}{s}$$

Aufgabe Nr. 4

$E = mc^2$

Elektron = Kugelkondensator

$$U = \int_r^\infty E\,dr' = \int_r^\infty \frac{1}{4\pi\varepsilon_0}\frac{-e}{r'^2}\,dr'$$

$$= \frac{-e}{4\pi\varepsilon_0}\int_r^\infty \frac{1}{r'^2}\,dr' = \frac{-e}{4\pi\varepsilon_0}\left.\frac{-1}{r}\right|_r^\infty$$

$$= \frac{-e}{4\pi\varepsilon_0}\left[0 + \frac{1}{r}\right] = \frac{-e}{4\pi\varepsilon_0}\frac{1}{r_0}$$

$$\Rightarrow E = \frac{1}{2}\,\phi U = mc^2 = \frac{1}{2}(-e)\frac{(-e)}{4\pi\varepsilon_0}\frac{1}{r_0}$$

$$\boxed{r_0 = \frac{e^2}{8\pi\varepsilon_0 mc^2}}$$

$$= 1{,}41 \cdot 10^{-15}\ \mathrm{m}$$

Aufgabe 14.5

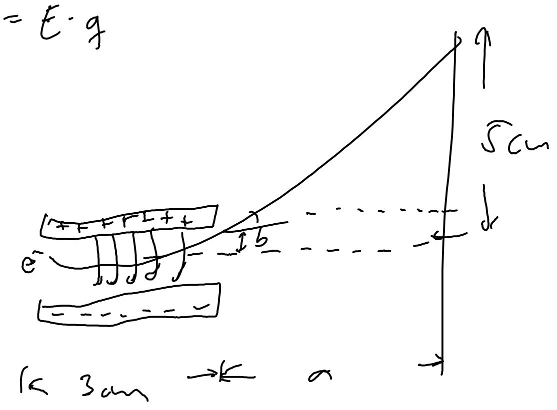
a) $W = U q$

$W = W_{kin} = \frac{1}{2} m_e v_0^2$

$x = v_0 \cdot t$ mit $F_{el} = m_e \cdot a = E \cdot q$

$y = \frac{1}{2} a t^2$

$\Rightarrow y = \frac{1}{4} \frac{E}{U} x^2$



$y = \frac{1}{2} \frac{e}{m} E \frac{x^2}{v_0^2}$ | $U \cdot e = \frac{1}{2} m v_0^2 \rightarrow v_0^2 = 2 \cdot U \frac{e}{m}$

$y = \frac{1}{2} \frac{e}{m} E \frac{x^2}{2U \frac{e}{m}} = \frac{1}{4U} E x^2$

b) $\left.\frac{dy}{dx}\right|_{x=3cm} \cdot a = 5cm$

$\frac{1}{2} \frac{E}{U} 3cm \cdot a = 5cm$

$E = 2 \frac{5}{3} \frac{U}{a} = 10 \frac{kV}{m} \left(9{,}7 \frac{kV}{m} \right)$

Aufgabe 15.1

a) $Q_0 = Q_1$

$$C_1 = \varepsilon \frac{A}{4d} = \frac{C_0}{4}$$

$$U_1 = \frac{Q_0}{C_1} = 4 \frac{Q_0}{C_0} = 4U_0$$

$$E_1 = \frac{U_1}{4d} = \frac{U_0}{d} = E_0$$

b) $W_0 = \frac{1}{2} Q_0 U_0$

$$W_1 = \frac{1}{2} Q_1 U_1 = \frac{1}{2} Q_0 4U_0$$

$$W = W_1 - W_0 = \frac{3}{2} Q_0 U_0$$

c) $U_0 = U_1$

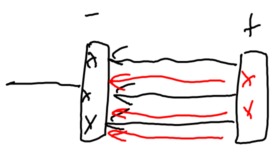
$$C_1 = \frac{C_0}{4}$$

$$Q_1 = C_1 U_1 = \frac{Q_0}{4}$$

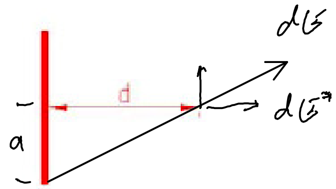
$$E_1 = \frac{U_1}{4d} = \frac{E_0}{4}$$

d) $F_{el} = E_i \, Q = \frac{U^2 C}{2d} = 55{,}3\,mN$

$$E_1 + E_2 = \frac{U}{d}$$



Aufgabe 15.2



$$dE^* = dE \cdot \frac{d}{\sqrt{a^2+d^2}}$$

$$= \frac{dQ \cdot d}{\varepsilon_0 4\pi (a^2+d^2) \sqrt{a^2+d^2}}$$

$$\rho_L = \frac{Q}{2\pi a} \qquad dQ = \rho_L \cdot dx$$

$$E^* = \int_0^{2\pi a} \frac{\rho \cdot d}{4\pi\varepsilon_0 (a^2+d^2)^{\frac{3}{2}}} dx = \frac{Q \cdot d}{4\pi\varepsilon_0 (a^2+d^2)^{\frac{3}{2}}}$$

a) $E = 0 \frac{V}{m}$

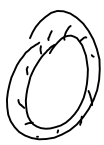
b) $E = E^*$

$$\varphi = -\int_{-\infty}^{d} E\,dx \qquad x^2 = y \quad dy = 2x\,dx$$

$$= -\frac{Q}{8\pi\varepsilon_0} \int_{-\infty}^{d} \frac{2x}{(a^2+x^2)^{\frac{3}{2}}} dx$$

$$= -\frac{Q}{8\pi\varepsilon_0} \int_{\infty}^{d^2} \frac{1}{(a^2+y)^{\frac{3}{2}}} dy \qquad \varphi = \frac{Q}{4\pi\varepsilon_0\sqrt{a^2+d^2}}$$

c) $dE^* = \frac{dQ \cdot d}{4\pi\varepsilon_0 (a^2+d^2)^{\frac{3}{2}}}$; $\rho = \frac{Q}{\pi a^2}$ $\quad dQ = \rho \cdot dA$



$$dA = 2\pi \cdot a \cdot da$$

$$E^* = \int_0^{r} \frac{\rho \cdot 2\pi \cdot a d}{4\pi\varepsilon_0 (a^2+d^2)^{\frac{3}{2}}} da \ ; \ dQ = \rho \cdot dA$$

$$= \frac{Q}{2\pi\varepsilon_0 a^2} - \frac{Q d}{2\pi\varepsilon_0 a^2 \sqrt{d^2+a^2}}$$

$$\varphi = -\int_{\infty}^{d} \left(\frac{Q}{2\pi\varepsilon_0 a^2} - \frac{Q x}{2\pi\varepsilon_0 \sqrt{(x^2+a^2)}} \right) dx = \frac{Q}{2\pi\varepsilon_0 a^2} \cdot \left(\sqrt{d^2+a^2} - d \right)$$

$$E = -\frac{d\varphi}{dd}$$

Aufgabe 15.3

Coulombkraft = Reibungskraft

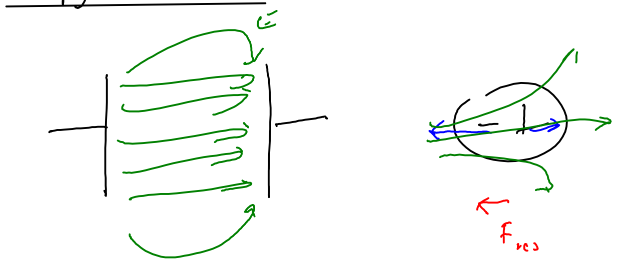
$$z \cdot e \cdot E = 6\pi \eta r v$$

$$\mu = \frac{v}{E} = \frac{ze}{6\pi \eta r}$$

$$r = 3{,}78 \times 10^{-10}\,m = 3{,}78\ \text{Å}$$

09.05.2008

Aufgabe 15.4



$$W_C = \frac{1}{2} Q U = \frac{1}{2} C U^2$$

$$W_{pot} = \frac{1}{2} m g h$$

$$W_{Batt} = Q U = C U^2$$

$$W_{Batt} = W_{pot} + W_C$$

$$0 = \frac{1}{2} \rho g s d h^2 - \frac{(\varepsilon_r - 1) h + H}{2d} \varepsilon_0 b U^2 = W_{ges}$$

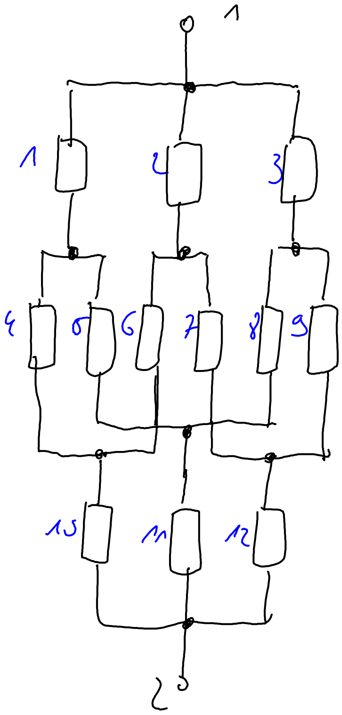
$$2 W_C = W_{pot} + W_C$$

$$0 = W_{pot} - W_C$$

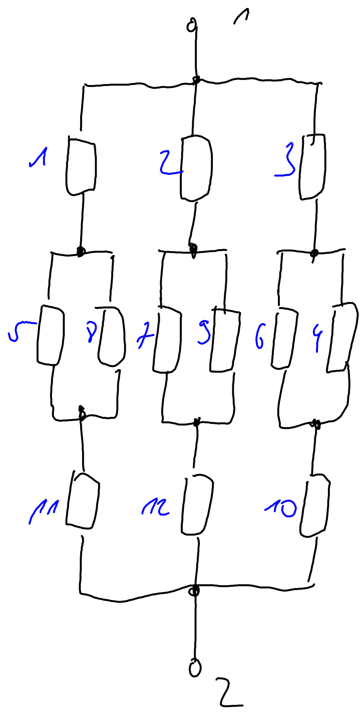
$$\frac{dW_{ges}}{dh} = 0 \Rightarrow h = \frac{(\varepsilon_r - 1)\varepsilon_0}{2 d^2 \rho g} U^2$$

$$\rightarrow \varepsilon_r = \frac{2 \rho d^2 g h}{U^2 \varepsilon_0} + 1$$

Aufgabe 15.5



da Widerstände auf gleichen => Potenziale liegen



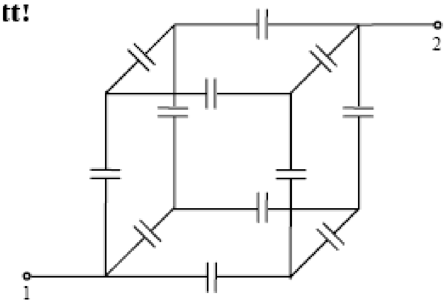
$$\frac{1}{R_{ges}} = \frac{1}{R_I} + \frac{1}{R_{II}} + \frac{1}{R_{III}} = \frac{1}{1\Omega + \frac{1\Omega^2}{2\Omega} + 1\Omega} + \frac{1}{2{,}5\Omega} + \frac{1}{2{,}5\Omega} = \frac{3}{2{,}5\Omega} = \frac{6}{5}\Omega$$

$$R_{ges} = \frac{5}{6}\Omega$$

Aufgabe 16.1

3 parallel + 6 parallel + 3 parallel

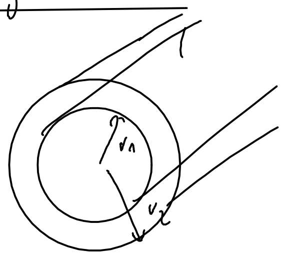
tt!



$$\frac{1}{C_{ges}} = \frac{1}{3C_1} + \frac{1}{6C_1} + \frac{1}{3C_1} = \frac{5}{6C_1}$$

$$C_{ges} = \frac{6}{5} C_1$$

Aufgabe 16.2



$$D = \frac{Q}{A} \Rightarrow D(r) = \frac{Q}{2\pi r l}$$

$$D = \varepsilon \cdot E \Rightarrow E(r) = \frac{Q}{2\pi \varepsilon_0 \varepsilon_r \cdot r \cdot l}$$

$$U_{ab} = \int_a^b E(r)\,dr = \frac{Q}{2\pi \varepsilon_0 \varepsilon_r l} \ln\left(\frac{b}{a}\right)$$

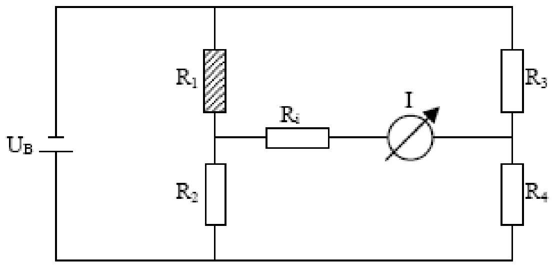
D.. Verschiebungsdichte

$$C = \frac{Q}{U} = \frac{2\pi \varepsilon_0 \varepsilon_r l}{\ln\left(\frac{b}{a}\right)}$$

$$= 126{,}5 \frac{pF}{m}$$

Aufgabe 16.3

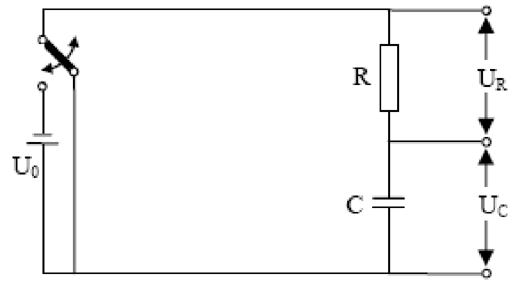
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$



$$R_1 = R_{20}\left(1+\alpha(\vartheta - 293{,}15K)\right) = \rho_{20}\frac{l}{A}\left(1+\alpha(\vartheta - 293{,}15K)\right)$$

$$\vartheta = \frac{R_2 R_3}{R_4 \rho_{20}\frac{l}{A}\alpha} - \frac{1}{\alpha} + 293{,}15K = 273{,}92K = 0{,}77\,°C$$

Aufgabe 16.4



Schalter geschlossen:

$$U_0 = U_R + U_C$$

$$U_0 = I \cdot R + \frac{Q}{C} \qquad I = \frac{dQ}{dt}$$

$$U_0 = \dot{Q} \cdot R + \frac{Q}{C} \qquad \underline{DGL}$$

$$\dot{Q} = \frac{U_0}{R} - \frac{Q}{CR} \qquad Q(t) = A \cdot e^{-\frac{1}{CR}t} + B$$

$$\dot{Q}(t) = -\frac{A}{CR} e^{-\frac{1}{CR}t}$$

Ansatz: $U_C = U_0\left(1 - e^{-\frac{t}{\tau}}\right)$

$$U_0 = U_C + U_R \; ; \; U_C = \frac{Q}{C} \rightarrow U_R = \frac{dQ}{dt} R = C\dot{U}_C \cdot R$$

$$U_0 = U_0\left(1 - e^{-\frac{t}{\tau}}\right) + CR\dot{U}_C$$

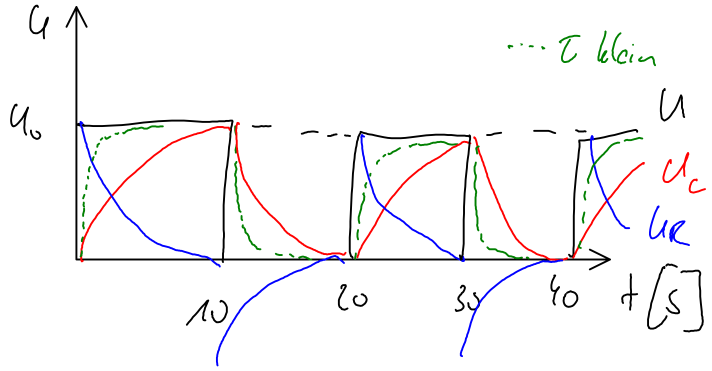
$$= U_0\left(1 - e^{-\frac{t}{\tau}}\right) + \frac{CR}{\tau} U_0 e^{-\frac{t}{\tau}}$$

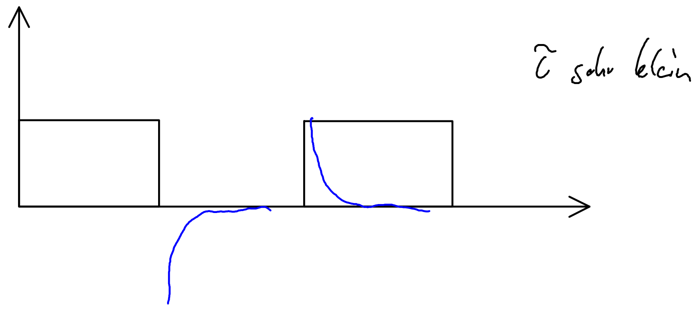
$$U_0 = U_0 - U_0 e^{-\frac{t}{\tau}} + \frac{CR}{\tau} U_0 e^{-\frac{t}{\tau}}$$

$$0 = e^{-\frac{t}{\tau}}\left(\frac{CR}{\tau} - 1\right) \rightarrow \boxed{\tau = CR}$$

$U_c = U_0 \left(1\ e^{-\frac{t}{\tau}}\right)$

$U_R = U_0 - U_c$





b) $U_R = IR = QR = \dot{Q}R = RC\dot{U}_c$

$$= RC \frac{d}{dt}\left(U_0 \left(1 - e^{-\frac{t}{\tau}}\right)\right)$$

$$U_R = RC\left[\dot{U}_0 \left(1 - \underbrace{e^{-\frac{t}{\tau}}}_{\approx 0}\right) + \underbrace{\frac{1}{\tau C} e^{-\frac{t}{\tau}}}_{\approx 0} U_0\right]$$

$$U_R \approx RC \dot{U}_0 = \tau \frac{dU_0}{dt} \qquad (*)$$

$U_c = U_0 - U_R$

$* \Rightarrow \approx U_0 - \tau U_0$ τ sehr groß

Tutor Email: phy51432@stud.uni-stuttgart.de

Aufgabe M.4

$$U_c = U_0\left(1 - e^{-\frac{t}{\tau}}\right) \quad ; \quad \tau = R \cdot C$$

$$U_R = I \cdot R = \dot{Q}_C R = RC\,\dot{U}_C = RC\left(1 - e^{-\frac{t}{\tau}}\right)$$

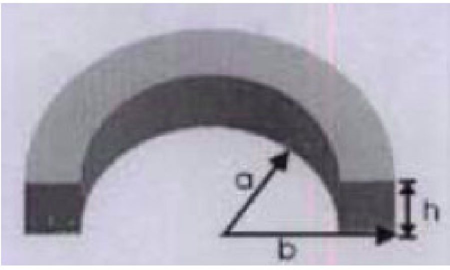
$$U_0 = U_R + U_C = \underbrace{RC}_{\tau}\,\dot{U}_C + U_C$$

$\tau = RC$ sehr groß

$$U_0 \approx RC\,\dot{U}_c \rightarrow \dot{U}_C = \frac{1}{RC} U_0$$

$$\underline{\underline{U_C(t) = \frac{1}{RC}\int_0^t U_0(t')\,dt'}}$$

Aufgabe 17.1



a) $R = \rho \cdot \frac{l}{A} \Rightarrow \rho \cdot \frac{\pi r}{h \cdot dr}$

$\frac{1}{R_{ges}} = \sum \frac{1}{R_i}$

$\Rightarrow \frac{1}{R} = \frac{h\,dr}{\rho \pi r}$

$\Rightarrow \int_a^b \frac{h\,dr}{\rho \pi r} = \frac{h}{\rho\pi} \cdot \ln r \Big|_a^b = \frac{h}{\rho\pi} \cdot \ln\left(\frac{b}{a}\right)$

b) $\rho = 1{,}78 \cdot 10^{-8}\,\Omega m$

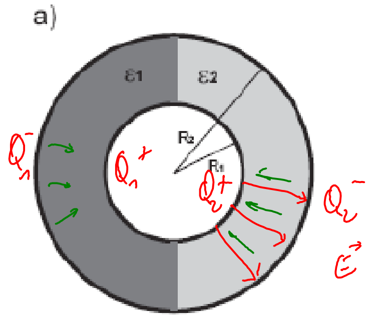
$\frac{1}{R} = \frac{h}{\rho\pi} \cdot \ln\left(\frac{b}{a}\right)$

$R = 3{,}96 \cdot 10^{-6}\,\Omega$

c) R steigt wegen Kristallgitter

30.05.2008

Aufgabe 17.2



a) $C = \dfrac{2\pi\varepsilon_0\varepsilon_r}{\frac{1}{R_i} - \frac{1}{R_a}}$ (Halbkugel)

$C_1 \parallel C_2 \rightarrow C_{ges} = C_1 + C_2 = 2\pi\varepsilon_0 \dfrac{\varepsilon_1+\varepsilon_2}{\frac{1}{R_1} - \frac{1}{R_2}}$

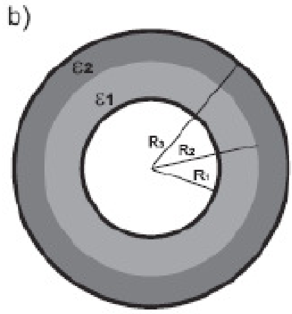
$U = \int \vec{E}\, d\vec{r}$

4mal (für $\varepsilon_1, \varepsilon_2, R_1, R_2$):

$$\sigma_{\varepsilon_1}^{R_1} = \frac{C^{\varepsilon_1} U}{A_{R_1}} = \frac{2\pi\varepsilon_0\varepsilon_1}{\frac{1}{R_1} - \frac{1}{R_2}} \frac{U}{2\pi R_1^2} = \underline{\underline{\frac{\varepsilon_0\varepsilon_1 U}{\left(1-\frac{R_1}{R_2}\right)R_1}}}$$

Andere Flächen analog!

b) $\dfrac{1}{C} = \dfrac{\frac{1}{R_1} - \frac{1}{R_2}}{4\pi\varepsilon_1\varepsilon_0} + \dfrac{\frac{1}{R_2} - \frac{1}{R_3}}{4\pi\varepsilon_2\varepsilon_0}$



$$\sigma = \frac{CU}{A} = \frac{4\pi\varepsilon_0}{\frac{1}{\varepsilon_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right) + \frac{1}{\varepsilon_2}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)} \cdot \frac{U}{4\pi R_i^2}$$

allgemeine Formel für alle R_i

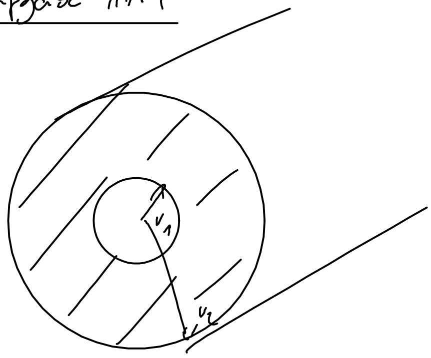
Aufgabe 17.3

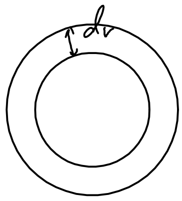
$$C = \varepsilon \frac{A}{d}$$

$$CU = Q \qquad C = 8{,}85 \cdot 10^{-12} F$$

$$F = \frac{QU}{2d} = \frac{CU^2}{2d} = \underline{\underline{0{,}44\ N}}$$

Aufgabe 17.9





$$dR = \rho \frac{dr}{2\pi r l} \quad \Rightarrow \quad R = \int_{r_1}^{r_2} \rho \frac{dr}{2\pi r l}$$

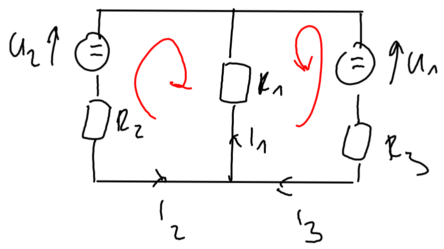
$$R = \rho \frac{d}{A}$$

$$R = \frac{\rho}{2\pi l} \ln\left(\frac{r_2}{r_1}\right)$$

$$R = \frac{10^{12}\,\Omega m}{2\pi \cdot 100\,m} \cdot \ln(8) = 3{,}3 \cdot 10^{3}\,\Omega$$

$$I = \frac{U}{R} = \frac{3 \cdot 10^{3}\,V}{3{,}3 \cdot 10^{3}\,\Omega} = 0{,}9\,\mu A$$

Aufgabe 18.1



LGS: $U_2 - U_{R_1} - U_{R_2} = 0$

$U_1 - U_{R_1} - U_{R_3} = 0$

$I_1 - I_2 - I_3 = 0$

$U_{R_1} = R_1 \cdot I_1$

$U_{R_2} = R_2 \cdot I_2$

$U_{R_3} = R_3 \cdot I_3$

$$\begin{pmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} U_2 \\ U_1 \\ 0 \end{pmatrix}$$

$I_1 = \frac{U_1 + U_2}{2R_1 + R_3} = 0{,}5\,A$

$I_2 = \frac{U_2 - R_1 I_1}{R_2} = 0{,}125\,A$

$I_3 = I_1 - I_2 = 0{,}375\,A$

Aufgabe 18.2

$$P = UI = \frac{U^2}{R}$$

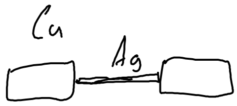
$$P_p = \frac{U^2 (R_1 + R_2)}{R_1 R_2}$$

$$P_R = \frac{U_2}{R_1 + R_2}$$

$$P_p = 6 P_R$$

$$R_1 = (2 \pm \sqrt{3})\, R_2$$

Aufgabe 18.3



$$P = \frac{W}{t} \qquad \frac{P}{c} = \frac{\Delta T \cdot m}{t}$$

$$c = \frac{W}{\Delta T \cdot m} \qquad \frac{P}{c \cdot m} = \frac{\Delta T}{t}$$

$$P = R I^2 = \rho_{R\,Ag} \cdot \frac{l_{Ag}}{A_{Ag}} \cdot I^2$$

$$m = \rho_{D\,Ag} \cdot l_{Ag} \cdot A_{Ag}$$

$$\frac{\Delta T}{t} = \frac{\rho_{R\,Ag} \frac{\cancel{l_{Ag}}}{A_{Ag}} I^2}{c_{Ag} \cdot \rho_{D\,Ag} \cancel{l_{Ag}} \cdot A_{Ag}}$$

$$= \frac{\rho_{R\,Ag} \cdot I^2}{c_{Ag} \cdot \rho_{D\,Ag} \cdot A^2_{Ag}}$$

$$\Delta T = 941\,°C$$

$$\frac{\Delta T}{\frac{\Delta T}{t}} = t = 0{,}22\,s$$

Aufgabe 18.4

$A_r = 63{,}55$

$4 A_r =: M_r$

$m = M_r \cdot \frac{1}{12} m_m^{12c} = 4{,}22 \cdot 10^{-25}\,kg$

$V = \frac{m}{\rho} = 4{,}73 \cdot 10^{-29}\,m^3$

$V = d^3$

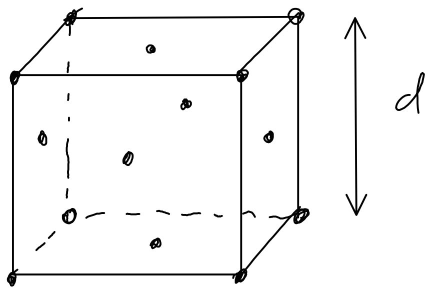
$d = 362\,pm$

pro Einheitszelle → 4 Elektronen

$n = \frac{4}{V}$

$j = nq v_0$

$|v_0| = \frac{j}{nq} = +5{,}91 \cdot 10^{-4}\,\frac{m}{s}$



$8\,\frac{1}{8} + 6\,\frac{1}{2} = 4$

Aufgabe 16.5

a) $qE = v \cdot q \cdot B_1$

$v = \frac{E}{B_1}$

$\frac{mv^2}{r} = vq \cdot B_2$

$\frac{mv}{r} = q \cdot B_2 \quad \Rightarrow r = \frac{mv}{qB_2}$

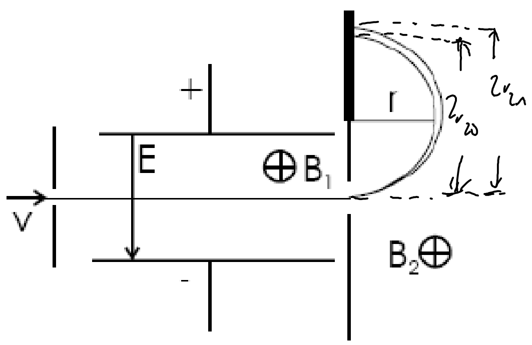
$\frac{q}{m} = \frac{v}{r \cdot B_2} = \frac{E}{r \cdot B_1 \cdot B_2}$

b) $B_1 = \frac{5 \cdot 10^5 Vs}{5 \cdot 10^6 m^2} = 0{,}1 T$

$B_2 = 0{,}1 T$

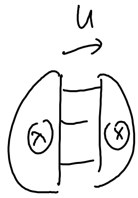
$^{20}Ne: r = \frac{3{,}951 \cdot 10^{-26} kg \cdot 5 \cdot 10^6 m}{e \cdot 0{,}1 T \quad s} = 0{,}1038 m$

$^{21}Ne: r = 0{,}1089 m$



Aufgabe 20.1

a)



$$F_Z = F_L$$

$$\frac{m v^{\cancel{2}}}{r} = q \cancel{v} B \qquad \omega = \frac{v}{r}$$

$$\omega = \frac{qB}{m}$$

$$\Rightarrow \omega_Z = 2{,}17 \cdot 10^7 \frac{1}{s}$$

$$\Rightarrow f = 3{,}45 \, MHz$$

$\Delta W = 23{,}5 \, MeV$ in 120 Durchläufen

$$\Delta W_1 = 2q \cdot U = 4 \cdot e \cdot U$$

$$\Delta W = 120 \cdot \Delta W_1$$

$$480 \, e \cdot x = 23{,}5 \cdot 10^6 \, eV$$

$$\Rightarrow \underline{\underline{x = 49 \, kV}}$$

b) $\frac{1}{2} m v_i^2 = W_i$

$$v_i = \sqrt{\frac{2W_i}{m}}$$

$$r_i = \frac{m v_i}{qB} = \frac{\sqrt{2W_i}}{2eB}$$

$$r_1 = 0{,}39 \, m$$

$$r_2 = 1{,}6 \, m$$

Aufgabe 20.2

a) $I_{ind} = \frac{U_{ind}}{R} = \frac{-\dot{A}B}{R} = \frac{Bbv}{R}$

b) $F_G = F_L$

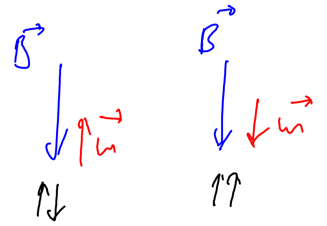
$mg = \frac{B^2 b^2 v}{R} \Rightarrow v = \frac{Rmg}{B^2 b^2}$

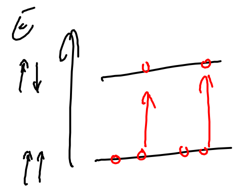
c) $I_{ind} = 0 \Rightarrow$ freier Fall

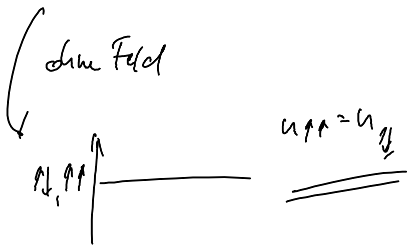
Aufgabe 20.3

$$\Delta n = n_{\uparrow\uparrow} - n_{\uparrow\downarrow} \qquad (1)$$

$$\frac{n_{\uparrow\downarrow}}{n_{\uparrow\uparrow}} = e^{-\frac{E_{\uparrow\downarrow} - E_{\uparrow\uparrow}}{k_B T}}$$







z.z. $\Delta E \ll k_B T \Rightarrow \chi \sim \frac{1}{T}$

$$n_{\uparrow\uparrow} + n_{\uparrow\downarrow} = n \qquad (1): \quad n_{\uparrow\downarrow} = \frac{n - \Delta n}{2} \; ; \; n_{\uparrow\uparrow} = \frac{n + \Delta n}{2}$$

$$E_{pot,m} = -\vec{m}\vec{B}$$

$$\frac{n_{\uparrow\downarrow}}{n_{\uparrow\uparrow}} = \frac{n - \Delta n}{n + \Delta n} = \exp\left(\frac{-E_{\uparrow\downarrow} - E_{\uparrow\uparrow}}{k_B T}\right) = \exp\left(-\frac{2\vec{m}\vec{B}}{k_B T}\right)$$

$$\Rightarrow \frac{\Delta n}{n} = \frac{1 - \exp\left(-\frac{2mB}{k_B T}\right)}{1 + \exp\left(-\frac{2mB}{k_B T}\right)} = \tanh\left(\frac{mB}{k_B T}\right)$$

$$\frac{mB}{k_B T} \ll 1 \; ; \quad \tanh\left(\frac{mB}{k_B T}\right) \overset{\text{Taylor-Reihe}}{\approx} \frac{mB}{k_B T}$$

Magnetisierung $\quad M = \Delta n \cdot m = \underbrace{\frac{n m^2}{k_B T}} B = \chi(T) \cdot B$

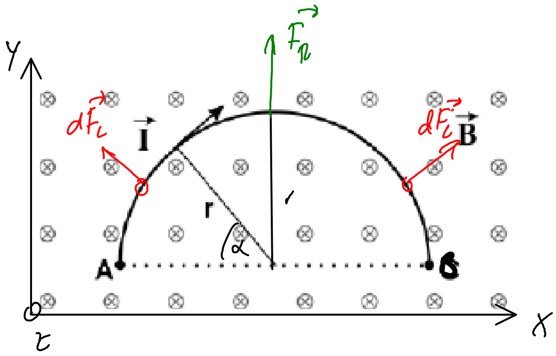
$$\underline{\underline{\chi(T) = \frac{C}{T}}} \qquad \square$$

Aufgabe 21.1

$$\left.\begin{array}{l} dF = B \cdot I \cdot dl \\ dl = R \cdot \sin\alpha \end{array}\right\} dF_x = B \cdot I \cdot R \cdot \sin\alpha$$

$$F = \int_0^\pi dF_x = 2BIR$$

$$d\vec{F_L} = I(d\vec{s} \times \vec{B})$$



$$d\vec{F_L} = d\vec{s}(\vec{I} \times \vec{B})$$

$$\vec{r}(\alpha) = \begin{pmatrix} -\cos\alpha \cdot r \\ \sin\alpha \cdot r \\ 0 \end{pmatrix} \quad d\vec{s} = \begin{pmatrix} \sin\alpha \cdot r \\ \cos\alpha \cdot r \\ 0 \end{pmatrix}$$

$$d\vec{F_L} = I \begin{pmatrix} \sin\alpha\, r \\ \cos\alpha\, r \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ -B \end{pmatrix}$$

$$\vec{F_L} = I \int_0^\pi \begin{pmatrix} -\cos\alpha\, r B \\ \sin\alpha\, r B \\ 0 \end{pmatrix} d\alpha = I \begin{pmatrix} 0 \\ 1+1 \\ 0 \end{pmatrix} r B \quad \underline{\underline{= 2 I r B\, \vec{e_y}}}$$

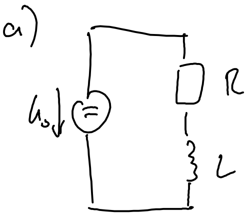
$$d\vec{s} = \frac{d\vec{r}(\alpha)}{d\alpha}$$

Gerades Leiterstück:

$$\vec{F_L} = l(\vec{I} \times \vec{B}) = 2r I B \vec{e_y}$$

Aufgabe 21.2

a)



$U_0 = U_R + U_L$

$U_0 = I \cdot R + L \dot{I}$

Ansatz: $I(t) = a \cdot e^{bt} + c$

$I(0) \overset{!}{=} 0 \Rightarrow 0 = a + c$

$\Rightarrow a = -c$

$t \to \infty \Rightarrow I(t) \Rightarrow \frac{U_0}{R}$

$\Rightarrow c = \frac{U_0}{R}$

$I(t) = \frac{U_0}{R}\left(1 - e^{bt}\right)$

$\dot{I}(t) = -\frac{U_0}{R} \cdot b \cdot e^{bt}$

$U_0 = U_0\left(1 - e^{bt}\right) - L \cdot \frac{U_0}{R} \cdot b\, e^{bt}$

$b = \frac{-R}{L} \Rightarrow I(t) = \frac{U_0}{R}\left(1 - e^{-\frac{R}{L} \cdot t}\right)$

b) $e^{-\frac{R}{L} \cdot t} \overset{!}{=} 0{,}99$

$-\frac{R}{L} \cdot t = \ln 0{,}99$

$t = -\frac{L}{R} \ln 0{,}99$

c) $U = L \cdot \dot{I}$

$= + \cancel{L} \cdot \frac{U_0}{\cancel{R}} \cdot \frac{\cancel{R}}{\cancel{L}} \cdot e^{-\frac{R}{L} \cdot t}$

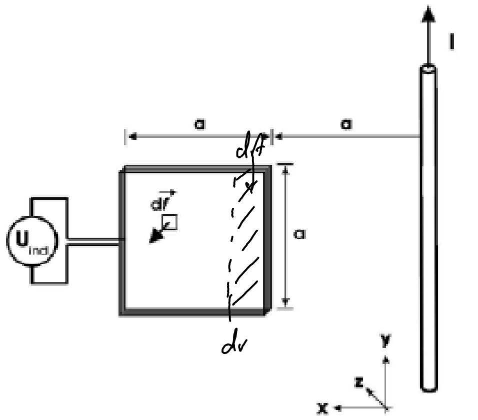
$\Rightarrow U(t) = U_0\, e^{-\frac{R}{L} \cdot t}$

Aufgabe 21.3

$$dU_{ind} = -N\left(\frac{d\phi}{dt}\right) = -N\,\frac{dB(r)}{dt}\cdot dA \quad ; \quad dA = dr\cdot a$$

$$B(r) = \mu_0\,\frac{I(t)}{2\pi r}$$

$$dU_{ind} = -N\,\frac{d}{dt}\left(\mu_0\,\frac{I(t)}{2\pi r}\,dA\right)$$



$$U_{ind} = -N\,\frac{d}{dt}\left(\mu_0\,\frac{I(t)}{2\pi}\int_a^{2a}\frac{a}{r}\,dr\right)$$

$$= -Na\,\frac{d}{dt}\left(\mu_0\,\frac{I(t)}{2\pi}\ln\left(\frac{2a}{a}\right)\right)$$

$$= -Na\,\mu_0\,\frac{I_0\,\omega\,\sin(\omega t)}{2\pi}\ln 2$$

$$= Na\,\mu_0\cdot\nu\cdot I_0\cdot\sin(2\pi\nu t)\ln 2$$

$$\hat{U}_{ind} = Na\,\mu_0 I_0\,\nu\,\ln 2 = 2{,}17\,V$$

Aufgabe 21.4

a) Skript S.77 :

$$B = \frac{\mu_r \mu_0 N I}{l + d \cdot \mu_r}$$

$$H_L = \frac{\mu_r N I}{l + d \cdot \mu_r}$$

b) $d\mu_r \rightarrow 0 \Rightarrow H_L = \frac{\mu_r N I}{l} \Rightarrow N = 360$

$$\frac{H_L}{2} = \frac{\mu_r N I}{l + d\mu_r} \Rightarrow d = 5 \cdot 10^{-3} \, m$$

c) $L = \frac{N^2}{R_m} \Rightarrow L(d) = \frac{N^2 \mu_0 \mu_r A}{l + \mu_r \cdot d}$

$$R_{m_S} = R_{m_{Eisen}} + R_{m_L}$$

d) $U_{ind} = -L \frac{dI}{dt} = -0{,}56 \; H \cdot 2 \cdot 10^3 \frac{A}{s} = -1{,}146 V$

$L = 0{,}56 H \quad (d = 0)$

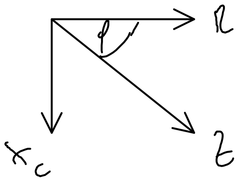
$$\frac{dI}{dt} = \frac{20 A}{10^{-2} s} = 2 \cdot 10^3 \; \frac{A}{s}$$

Aufgabe L2.3

a) $\underline{Z} = R - j\frac{1}{\omega C}$

$|\underline{Z}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = 1128{,}4\,\Omega$

$\omega = 2\pi f$



$\varphi = -\arctan\frac{1}{\omega C R} = -44{,}65°$

b) $I_{eff} = \frac{U_{eff}}{Z} = 194{,}97\,mA$

$U_R = R \cdot I_{eff} = 155{,}98\,V$

$P_L = U_R \cdot I_{eff} = 30{,}41\,W$

ohne C

$I_{eff} = \frac{U_{eff}}{R} = 275\,mA$

$U_R = U_{eff} = 220\,V$

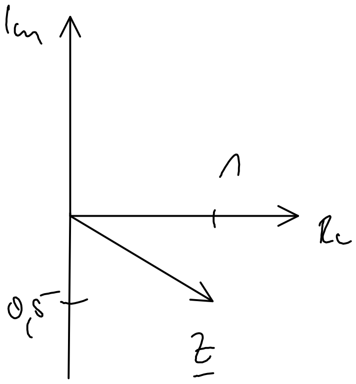
$P_2 = U_{eff} \cdot I_{eff} = 60{,}5\,W > P_L$

Aufgabe 23.1

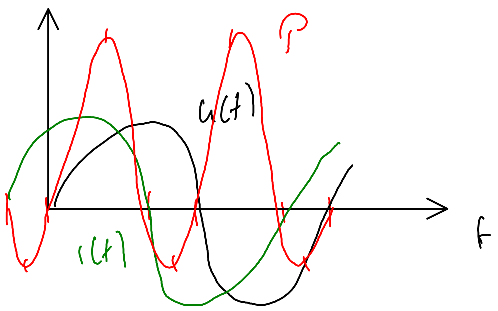
a) $\underline{Z} = R + j\left(L\omega - \frac{1}{\omega C}\right) = (1 - 0{,}5j)\ k\Omega$

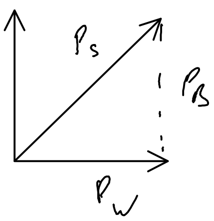
$\arg(\underline{Z}) = \arctan\left(-\frac{1}{2}\right)$

$= -0{,}46 \mathrel{\hat{=}} -26{,}6°$



b) $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{\hat{U} \cdot e^{j\varphi_u}}{\hat{I} \cdot e^{j\varphi_i}} = Z \cdot e^{j(\varphi_u - \varphi_i)}$





$P = \underline{U} \cdot \underline{I}^*$

c) $\mathrm{Im}(\underline{Z}) \overset{!}{=} 0 \Rightarrow L\omega = \frac{1}{\omega C}$

$L = \frac{1}{\omega^2 C} = 10\ H$

Aufgabe 23.3

$$\frac{\left(Z+j\omega\frac{L}{2}\right)\cdot\frac{1}{j\omega C}}{Z+j\omega\frac{L}{2}+\frac{1}{j\omega C}}+j\omega\frac{L}{2}=Z_{ges}=Z$$

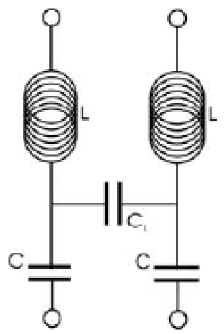
$$Z=\sqrt{\frac{L}{C}-\omega^2\frac{L^2}{4}}\quad\rightarrow\quad \Delta L \text{ und } \Delta C \text{ einsetzen}$$

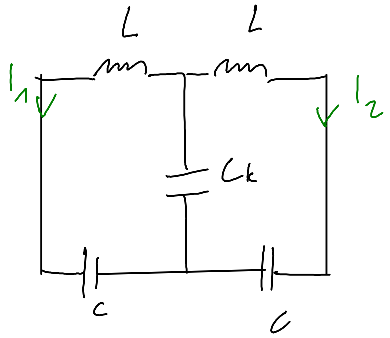
$$\Delta L\to 0\quad\Rightarrow\quad Z=\sqrt{\frac{\Delta L}{\Delta C}}$$

$$Z=\sqrt{\frac{\frac{\mu_0\mu}{2\pi}\,\Delta l\,\ln\left(\frac{r_a}{r_i}\right)}{2\pi\,\varepsilon_0\varepsilon_r\frac{\Delta l}{\ln\left(\frac{r_a}{r_i}\right)}}}\quad\Rightarrow\quad\frac{\ln\left(\frac{r_a}{r_i}\right)}{2\pi}\sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

Aufgabe 23.2

?





$$U_k = L\cdot \dot{I}_1 + \frac{Q_1}{C}$$

$$\dot{U}_k = L\cdot \ddot{I}_1 + \frac{I_1}{C} \longrightarrow -\frac{1}{C_k}(I_1+I_2) = L\ddot{I}_1 + \frac{I_1}{C}$$

$$0 = L\ddot{I}_1 + \left(\frac{1}{C}+\frac{1}{C_k}\right)I_1 + \frac{1}{C_k}I_2 \qquad (1)$$

$$0 = L\ddot{I}_2 + \left(\frac{1}{C}+\frac{1}{C_k}\right)I_2 + \frac{1}{C_k}I_1 \qquad (2)$$

Ansatz:

$$I_1 = \hat{I}_1 e^{i\omega t}$$

$$I_2 = \hat{I}_2 e^{i\omega t}$$

$$\Rightarrow 0 = \left(-L\omega^2 + \frac{1}{C} + \frac{1}{C_k}\right)I_1 + \frac{1}{C_k}I_2$$

$$0 = \left(-L\omega^2 + \frac{1}{C} + \frac{1}{C_k}\right)I_2 + \frac{1}{C_k}I_1$$

$$\begin{pmatrix} -L\omega^2 + \frac{1}{C} + \frac{1}{C_k} & \frac{1}{C_k} \\ \frac{1}{C_k} & -L\omega^2 + \frac{1}{C} + \frac{1}{C_k} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} \frac{1}{CL} + \frac{1}{C_kL} & \frac{1}{C_kL} \\ \frac{1}{C_kL} & \frac{1}{CL} + \frac{1}{C_kL} \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}}_{\vec{v}} = \underbrace{\omega^2}_{\alpha} \underbrace{\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}}_{\vec{v}}$$

$$M\vec{v} = \alpha\vec{v}$$

$$(M - \alpha E_2)\vec{v} = 0$$

$$|M - \alpha E_2| \overset{!}{=} 0$$

$$\left(\frac{1}{CL} + \frac{1}{C_k L} - \alpha\right)^2 - \left(\frac{1}{C_k L}\right)^2 = 0$$

$$\left(\frac{(C_k + C)L}{C_k C L^2} - \alpha\right)^2 - \left(\frac{1}{C_k L}\right)^2 = 0$$

$$\frac{C_k + C}{C_k C L} - \alpha = \frac{1}{C_k L}$$

$$\alpha = \frac{C_k + C}{C_k C L} - \frac{1}{C_k L}$$

$$\underline{\underline{\omega_1^2}} = \alpha = \frac{C_k}{C_k C L} = \underline{\underline{\frac{1}{CL}}}$$

$$\alpha = \frac{C_k + C}{C_k C L} + \frac{1}{C_k L}$$

$$\rightarrow \underline{\underline{\omega_2^2}} = \frac{2C + C_k}{C_k C L} = \underline{\underline{\frac{2}{C_k L} + \frac{1}{CL}}}$$

Aufgabe 24.1

$$\nabla^2 \vec{E}(\vec{r},t) - \varepsilon_0 \mu_0 \ddot{\vec{E}}(\vec{r},t) = 0$$

a) $E_z = E_0 \exp(i\omega t - ikz)$

$-k^2 E_z + \varepsilon_0 \mu_0 \omega^2 E_z = 0$

$-\left(\frac{\omega}{c_0}\right)^2 E_z + \varepsilon_0 \mu_0 \omega^2 E_z = 0$ $\quad c_0 = \sqrt{\frac{1}{\varepsilon_0 \mu_0}}$

$-\omega^2 \varepsilon_0 \mu_0 E_z + \varepsilon_0 \mu_0 \omega^2 E_z = 0$

$\square$

b) $E_\rho = \frac{E_0}{\rho} \exp(i(\omega t - k\rho))$

$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho^2 \frac{\partial}{\partial \rho} \left(\frac{E_0}{\rho} \exp(i\omega t - ik\rho) \right) \right) - \varepsilon_0 \mu_0 \ddot{E}_\rho$

$= -k^2 E_\rho + \varepsilon_0 \mu_0 \omega^2 E_\rho$

$= -\omega^2 \varepsilon_0 \mu_0 E_\rho + \omega^2 \varepsilon_0 \mu_0 E_\rho = 0$

$\square$

$\Delta = \underline{\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right)} + \frac{1}{\sin\vartheta\, r^2} \frac{\partial}{\partial \vartheta} \left(\sin\vartheta \frac{\partial}{\partial \vartheta} \right) + \frac{1}{\sin^2\vartheta\, r^2} \frac{\partial^2}{\partial \varphi^2}$ $\quad E = E(r)$

$= \frac{2}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial r^2}$

Aufgabe 24.2

a) $I = 1{,}4 \frac{kW}{m^2}$

$\bar{I} = \frac{1}{2} E \cdot H$

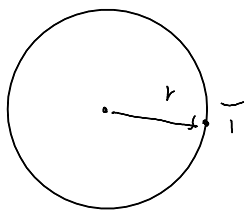
$ED = BH$

$$\varepsilon_0 E^2 = \mu_0 H^2 \Rightarrow \underset{\substack{\uparrow \\ \text{I quer}}}{I} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E^2$$

$$\Rightarrow E = \frac{1027}{\sqrt{2}} \frac{V}{m} \; ; \; H = \frac{2{,}72}{\sqrt{2}} \frac{A}{m} \Rightarrow B = \frac{2{,}72}{\sqrt{2}} \cdot 4\pi \cdot 10^{-7} T$$

$$= 2{,}4 \cdot 10^{-6} T$$

b)



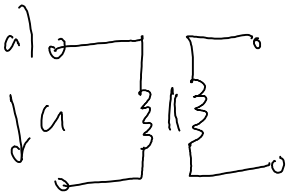
$$P = A \cdot \bar{I} = 4\pi r^2 \cdot \bar{I} = \underline{3{,}96 \cdot 10^{26} W}$$

c) $E = \frac{2{,}3}{\sqrt{2}} \cdot 10^{5} \frac{V}{m} = 1{,}6 \cdot 10^{5} \frac{V}{m}$

$H = \frac{609{,}6}{\sqrt{2}} \frac{A}{m}$

$$B = \frac{609{,}6 \cdot 4\pi \cdot 10^{-7}}{\sqrt{2}} T = 5{,}42 \cdot 10^{-4} T$$

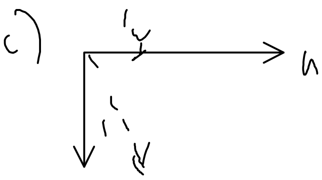
Aufgabe 24.3

a) 

$\underline{Z} = j\omega L$ da keine Last

b) $U_S = U_P \cdot \frac{n_S}{n_P}$

$P_{WS} = I_S \cdot U_S = \frac{U_S^2}{R_V}$ $\qquad I_S = \frac{U_P \cdot n_S^2}{R_V \cdot n_P^2}$

c) 

zu b) $\Delta I_P n_P = I_S n_S \rightarrow \Delta I_P = \frac{n_S^2}{n_P^2} \cdot \frac{U_P}{R_V}$

c) $I_P = \frac{U}{i\omega L_P} + \frac{n_S^2}{n_P^2} \frac{U_P}{R_V} \xrightarrow{\omega\to\infty} \underline{\underline{\frac{n_S^2}{n_P^2} \frac{U_P}{R_V}}}$

Aufgabe 22.2

1. Lsg $\vec{F} = \vec{m} \cdot \nabla \vec{B} \iff F_x = \vec{m} \cdot \nabla B_x$ (y, z analog)

$r = \sqrt{x^2 + y^2}$

2. Lsg

$$\vec{F} = \text{grad}(\vec{m} \cdot \vec{B}) - (\vec{m} \cdot \nabla)\vec{B} + \underbrace{(\vec{B} \cdot \nabla)\vec{m}}_{0} + \vec{m} \times (\nabla \times \vec{B}) + \underbrace{\vec{B} \times (\nabla \times \vec{m})}_{0}$$

$\vec{m} = \cos \omega t$