Axel Kabisch

$$2^0+2^1+2^2+\dots+2^{10} = 2^{11}-1$$

$$(x^5-1):(x-1) = x^4+x^3+x^2+x+1$$
$$\underline{-(x^5-x^4)}$$
$$x^4-1$$
$$\underline{-(x^4-x^3)}$$
$$x^3-1$$
$$\underline{-(x^3-x^2)}$$
$$x^2-1$$
$$\underline{-(x^2-x)}$$
$$x-1$$
$$\underline{-(x-1)}$$
$$0$$

$1+x=\frac{x^2-1}{x-1}$ 3. bin. Formel

$$\boxed{x^0+x^1+x^2+\dots+x^{10}=\frac{x^{11}-1}{x-1}}$$ endl. geometrische Summe

Binom. Formel

$$41^2=(40+1)^2=a^2+2ab+b^2$$
$$=40^2+2\cdot 40\cdot 1+1=1681$$

$$39^2=(40-1)^2=40^2-80+1=1521$$
$$a^2-2ab+b^2$$

endl. geom. Summe

$$-1+2-4+8\mp\dots\pm 2^{10}\pm 2^{11}$$
$$=\underbrace{-2^0+2^1}\underbrace{-2^2+2^3}-\dots\underbrace{-2^{10}+2^{11}}$$
$$=2^0+2^2+\dots+2^{10}$$

$2^n=\frac{2^{n+1}}{2}$

$$=(2^2)^0+(2^2)^1+\dots+(2^2)^5$$
$$=\frac{(2^2)^6-1}{2^2-1}=\frac{2^{12}-1}{3}=\frac{4096-1}{3}=1365$$

$$1+2+3+\ldots+100 = (100+1)+(99+2)+\ldots+(50+51)$$

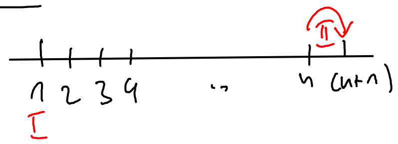
$$= 50 \cdot 101 = 5050$$

allgemeine Formel: $\forall\, n \in \mathbb{N}$:

$$1+2+\ldots+n = \frac{n}{2}(n+1)$$

Prinzip der vollständigen Induktion

Idee



Beweis für n und n+1 ✓

Beweis

zu I: Induktionsanfang $n=1$

$$1 \overset{\checkmark}{=} \frac{1}{2}(1+1) \overset{\checkmark}{=} \frac{n}{2}(n+1) \quad \checkmark$$

$$= \left(\frac{1}{2}\cdot 2\right)$$

zu II: Induktionsschritt von $n \to n+1$

gelte die Formel für ein festes n

zu zeigen: die Formel gilt für das "nächste" $\hat{=} (n+1)$

aus $A(n) \overset{?}{\Rightarrow} A(n+1)$

Voraussetzung:

$A(n)$ gilt: $1+2+\ldots+n = \frac{n}{2}(n+1)$ ✓ für ein festes n

Behauptung: $1+2+\ldots+n+(n+1) \overset{?}{=} \frac{n+1}{2}\left((n+1)+1\right)$

$$\frac{n+1}{2}(n+2) = \frac{n^2+2n+1n+2}{2}$$

$$\Downarrow$$

Bew: $n \to n+1$

$$1+2+3+\ldots+n+(n+1)$$

Vss

$$= \frac{n}{2}(n+1)+(n+1)$$

$$= \frac{n(n+1)}{2} + \frac{(n+1)2}{2} = \frac{n^2+n}{2} + \frac{2n+2}{2} = \frac{n^2+2n+1n+2}{2} = \frac{n+1}{2}(n+2) = \frac{(n+1)}{2}\left((n+1)+1\right) \quad \square$$

$$\frac{\partial f}{\partial x}(x) = f'(x)$$

$$f(x,y) = (x-3y)^2 + (3x-y)^2$$

$$\frac{\partial f}{\partial x} = 2\cdot(x-3y)\cdot 1 + 2(3x-y)\cdot 3 \qquad \frac{\partial f}{\partial y} = 2(x-3y)\cdot(-3) + 2(3x-y)\cdot(-1)$$

$$\int f'\cdot g\,dx = f(x)\cdot g(x) - \int f(x)\,g'(x)\,dx$$ Produktintegration

$$\int \underbrace{\sin(x)}_{f'}\,\underbrace{\sin(x)}_{g}\,dx$$

$g(x) = \sin x \qquad g'(x) = \cos x$

$f(x) = -\cos x \qquad f'(x) = \sin x$

$$= -\cos x \sin x - \int -\cos x \cos x\,dx$$

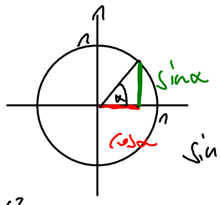
$$= -\cos x \sin x + \int \cos^2 x\,dx$$

$$= -\cos x \sin x + \int 1 - \sin^2 x\,dx$$

$$= -\cos x \sin x + \int 1\,dx - \int \sin^2 x\,dx \qquad \Big|+\int \sin^2 dx$$

$$2\cdot\int \sin^2 x\,dx = -\cos x \sin x + x$$

$\sin^2 x + \cos^2 x = 1$



$a^2 + b^2 = c^2$

$$\sin\alpha = \frac{\text{Gegenk.}}{\text{Hypoth.}}, \quad \text{Hypoth.} = 1$$

$$\cos\alpha = \frac{\text{Ankath.}}{\text{Hypoth.}}, \quad \text{Hypoth.} = 1$$

Integral jetzt leichter

$$\int x^2 e^x\,dx = e^x\cdot x^2 - \int e^x\cdot 2x\,dx = e^x\cdot x^2 - \left[2xe^x - 2e^x\right]$$

$g(x) = x^2 \qquad g'(x) = 2x$

$f(x) = e^x \qquad f'(x) = e^x$

$$\int e^x\cdot 2x\,dx = 2xe^x - \int 2e^x\,dx = 2xe^x - 2e^x$$

$g(x) = 2x \qquad g'(x) = 2$

$f(x) = e^x \qquad f'(x) = e^x$

DGL 1. Ordnung

$y' = 5x^4 \cdot (y+1)$ $\Rightarrow$ Löse die DGL, finde eine Fkt $y(t)$

$y'(x) = g(x) \cdot (y(x)+1)$

$y' = y \Rightarrow y' - y = 0$

$e^x = e^x$

$\frac{y'}{y+1} = 5x^4$

$\frac{1}{y+1} \frac{dy}{dx} = 5x^4$

$\frac{1}{y+1} dy = 5x^4 dx$ Trennung der Variablen

$\int \frac{1}{y+1} dy = \int 5x^4 dx$

$\ln(y+1) = x^5$

$e^{(\ln(y+1))} = e^{x^5}$

$y+1 = e^{x^5}$

$\underline{\underline{y = e^{x^5} - 1}}$

DGL 2. Ordnung

$y'' + y' - 2y = \underbrace{x}_{= g(x) \neq 0}$ (inhomogene DGL)

allg. Lösung der DGL

$y = y_0 + y_p$

y_0 ist Lsg der homog. DGL $y'' + y' - 2y = 0$

y_p ist eine spezielle Lsg. der inhomog. DGL

Berechnung y_0

charakteristische Gleichung $\lambda^2 + \lambda - 2 = 0$ $\boxed{\lambda_1 = 1}$ $x^2 + x - 2 = 0$

$\lambda_2 = -2$

$\boxed{y_0 = e^{1x}}$ $\boxed{y_0 = e^{-2x}}$

$y' = -2e^{-2x}$

$y'' = +4e^{-2x}$

oben einsetzen

$4e^{-2x} + (-2e^{-2x}) - 2e^{-2x} = 0$ $\rightarrow$ homogene Lsg

Papier falten so hoch wie Eiffelturm 300m

Papierdicke 0,3mm

$300m = 2^x \cdot 0{,}3\,mm$

$\cancel{3} \cdot 10^2 = 2^x \cdot \cancel{3} \cdot 10^{-4} m$

$10^6 = 2^x$

$10^3 = 1000$

$\approx 2^{10}$

$10^6 \approx 2^{20}$

$2^{20} = 2^x$

$\Rightarrow \underline{\underline{x = 20}}$

$2 \cdot 2^{(2x+4)} = 3 \cdot 2^{(x+2)} - 1$

$\boxed{x = ?}$

Subst: $z = 2^{x+2}$

$2^{(2x+4)} = 2^{((x+2)2)} = \left[2^{(x+2)}\right]^2$

$(a^m)^k = a^{m \cdot k}$

$\rightarrow 2z^2 = 3z - 1$

$2z^2 - 3z + 1 = 0$

$z^2 - \frac{3}{2}z + \frac{1}{2} = 0$

$z_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$

$= +\frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{8}{16}}$

$z_1 = 1$

$z_2 = 1/2$

$\rightarrow 1 = z = 2^{x+2}$

$0 = x + 2$

$\underline{\underline{x = -2}}$

$\frac{1}{2} = 2^{x+2}$

$-1 = x + 2$

$\underline{\underline{x = -3}}$

$$4^x - 24 \cdot 2^x + 128 = 0$$

$$z = 2^x$$

$$z^2 - 24z + 128 = 0$$

$$z_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

$$4^x = (2^2)^x = 2^{(2x)} = 2^{(x2)} = (2^x)^2$$

$$= 12 \pm \sqrt{144 - 128}$$

$$= 12 \pm 4$$

$$z_1 = 8 \qquad z_2 = 16$$

$$2^x = 8 \qquad 2^x = 16$$

$$x = 3 \qquad x = 4$$

$$4^{2x} - 20 \cdot 4^x + 64 = 0$$

$$4^{2x} - 20 \cdot 2^{2x} + 64 = 0$$

$$z = 2^{2x}$$

$$z^2 - 20z + 64 = 0$$

$$z_{1,2} = +10 \pm \sqrt{100 - 64}$$

$$z_1 = 16$$

$$z_2 = 4$$

$$2^{2x} = 16 \qquad 2^{2x} = 4$$

$$4^x = 16 \qquad 4^x = 4$$

$$x = 2 \qquad x = 1$$

$$\frac{1}{3} \cdot 4^{(3x-5)} = 5^{(2x-1)} \qquad | \log$$

$$\log\left(\frac{1}{3}\right) + \log\left(4^{3x-5}\right) = \log\left(5^{2x-1}\right)$$

$$\log(a \cdot b) = \log(a) + \log(b)$$

$$\log\frac{1}{3} + (3x-5) \cdot \log 4 = (2x-1) \cdot \log 5$$

$$\log(a^k) = k \cdot \log(a)$$

$$\log\frac{1}{3} + 3x\log 4 - 5\log 4 = 2x\log 5 - \log 5$$

$$3x\log 4 - 2x\log 5 = -\log\frac{1}{3} + 5\log 4 - \log 5$$

$$x(3\log 4 - 2\log 5) = \ldots$$

$$x = \frac{-\log\frac{1}{3} + 5\log 4 - \log 5}{3\log 4 - 2\log 5} \approx 6{,}8$$

$$9x^4 + 12x^3 = 3x^3(3x+4)$$

$$x^4 - 81 = (x^2-9)(x^2+9)$$

$$\downarrow \qquad\qquad \downarrow$$

$$(x-3)(x+3)(x^2+9)$$

$$\frac{3xy-y^2}{x^2-2xy+y^2} : \frac{y^2}{x-y} + \frac{x-3y}{x^2-y^2} \cdot \frac{2x+2y}{2x}$$

$$\frac{(3xy-y^2)(x-y)}{(x^2-2xy+y^2)y^2} + \frac{(x-3y)(2x+2y)}{2x(x^2-y^2)} = \frac{3x^2y \overset{4}{-\cancel{3xy^2} - \cancel{xy^2}} + y^3}{x^2y^2-2xy^3+y^4} + \frac{2x^2 \overset{-4}{+\cancel{2xy} - \cancel{6xy}} - 6y^2}{2x^3-2xy^2}$$

$$= \frac{\cancel{y}(3x^2-4xy+y^2)}{\cancel{y}(x^2y-2xy^2+y^3)} + \frac{\cancel{2}x^2 - \overset{2}{\cancel{4}}xy - \overset{3}{\cancel{6}}y^2}{\cancel{2}x^3 - \cancel{2}xy^2}$$

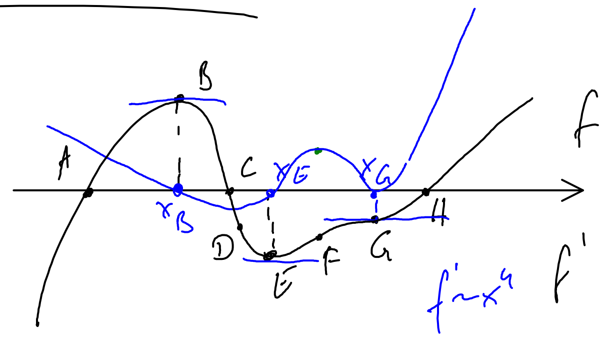
$$= \frac{(x^3-xy^2)(3x^2-4xy+y^2) + (x^2-2xy-3y^2)(x^2y-2xy^2+y^3)}{(x^2y-2xy^2+y^3)(x^3-xy^2)}$$

$$= \frac{3x^5 - 4x^4y + x^3y^2 - 3x^3y^2 + 4x^2y^3 - xy^4 + x^4y - 2x^3y^2 + x^2y^3 - 2x^3y^2 + 4x^2y^3 - 2xy^4}{\ldots}$$

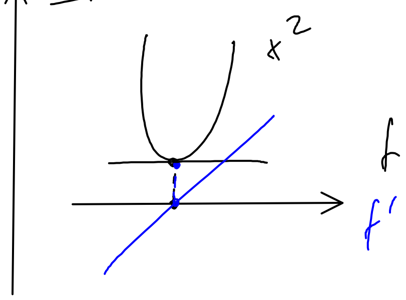
$$\frac{-3x^2y^3 + 6xy^4 - 3y^5}{\ldots}$$

$$= \frac{3x^5 - 3x^4y - 6x^3y^2 + 6x^2y^3 + 3xy^4 - 3y^5}{\ldots}$$

Kurvendiskussion



Bsp.:



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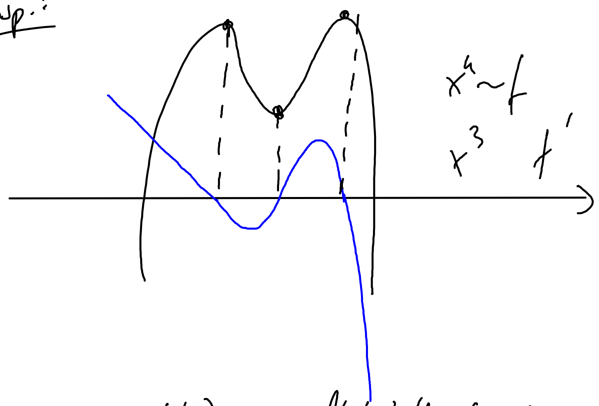
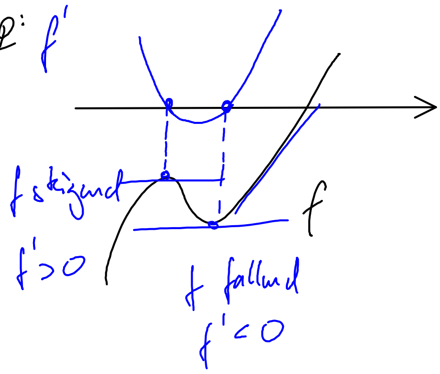


Bsp.: f'



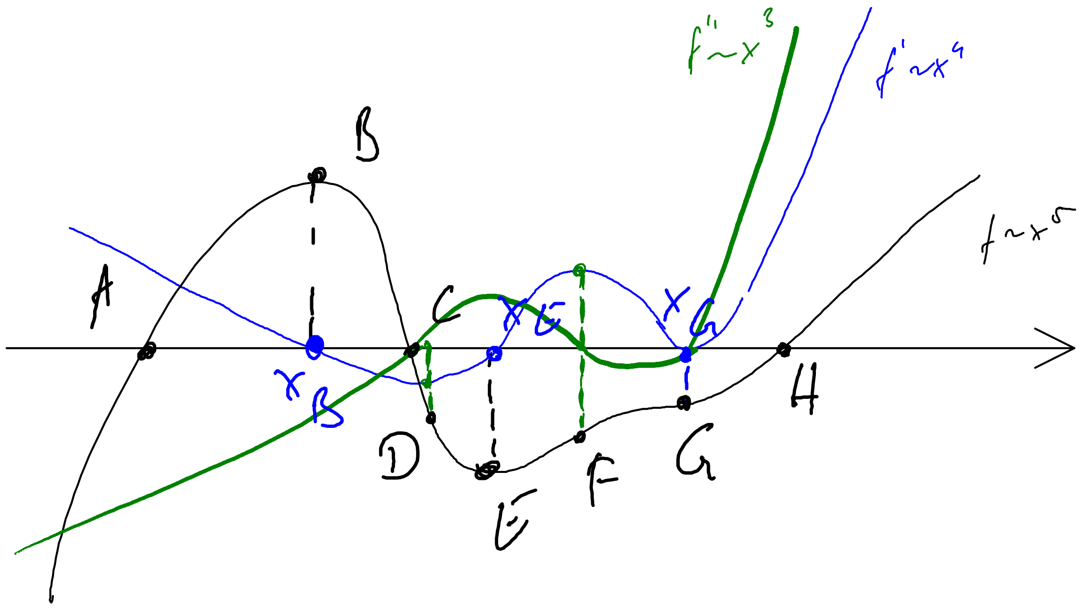
A, C, H : $f(x) = 0$ Nullstellen berechnen

G, B, E : $f'(x) = 0$ mögliche Extremstellen + Zusatzpunkte, Sattelpunkte

B, E : (1) $f'(x_B) = f'(x_E) = 0$

(2) $f(x_B) \overset{<0}{>} 0 \Rightarrow x_B$ Minimum Maxistelle

(2') VZW an der Stelle x_E von f' von $\overset{+}{-} \overset{\to -}{\to +}$



$f = \frac{u}{v}$

$f' = \frac{u'v - v'u}{v^2}$

$u = x-3 \qquad u' = 1$

$v = x^2-5 \qquad v' = 2x$

$f(x) = \frac{x-3}{x^2-5}$

$f' = \frac{1(x^2-5) - 2x(x-3)}{v^2}$

$f'(x) = 0 = x^2 - 5 - 2x^2 + 6x$

$= -x^2 + 6x - 5$

$\hookrightarrow x_1 = 1 \quad x_2 = 5$

21.10.2009

$f(x) = e^x \cdot (e^x - 2)$

1) Def.-Menge $\qquad D = \mathbb{R}$

2) Wertemenge $\qquad$ später

3) Schnittpunkte mit Achsen

4) EP und WP

5) Monotonieverhalten

3) $x = 0 \qquad \rightarrow (0|-1)$

$f(0) = e^0(e^0-2) = 1(1-2) = 1(-1) = -1$

Nullstellen von f:

$f(x) = 0 = e^x(e^x-2)$

$e^x = 2$

$x = \ln 2 \qquad \rightarrow S(\ln 2 / 0)$

4) EP:

$u = e^x \qquad u' = e^x$

$v = e^x - 2 \qquad v' = e^x$

$f'(x) = uv' + u'v$

$= (e^x)^2 + e^x(e^x-2) = (e^x)^2 + (e^x)^2 - 2e^x$

$= 2(e^{2x} - e^x)$

$= 2e^x(e^x-1)$

$x = 0$ mögl. Extremstelle

<u>Minimum</u> $(0|-1)$

EP:

$u = 2e^x \quad v = e^x - 1$

$u' = 2e^x \quad v' = e^x$

$f''(x) = 2e^x \cdot e^x + 2e^x \cdot (e^x - 1)$

$f''(0) = 2 \cdot 1 \cdot 1 + 2 \cdot 1 \cdot (0) = 2 \rightarrow > 0 \Rightarrow$ Minimum $(0 | -1)$

<u>VZW</u> von $f'(x)$:

$f'(-1) = 2e^{-1}(e^{-1} - 1) = \frac{2}{e}\left(\frac{1}{e} - 1\right) \underline{< 0}$

$f'(1) = 2e^1(e^1 - 1) \underline{> 0}$

$\rightarrow$ VZW von f' an der Stelle $x_E = 0$ von $- \rightarrow +$

WP:

$f''(x) = 2e^x e^x + 2e^x(e^x - 1)$

$= 2e^x(e^x + e^x - 1) = 2e^x(2e^x - 1) = 0$

$2e^x = 1$

$e^x = \frac{1}{2}$

$x = \ln\frac{1}{2} = \ln 1 - \ln 2 = -\ln 2 \approx -0{,}7 = x_W$

<u>Hinreichende Bedingung</u> für W-Pkt

f'' hat an der Stelle x_W einen VZW

<u>Prüfe</u> $x = -1$

$f''(-1) = \frac{2}{e}\left(\frac{2}{e} - 1\right)$

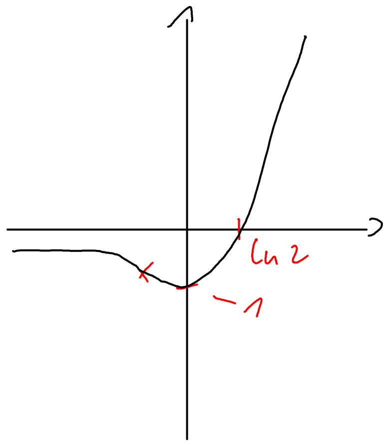
< 0

$x = 0$

$f''(0) = 2(2-1) = 2$

> 0

$\Rightarrow$ Wendepunkt $\left(-\ln 2 \,\middle|\, -\frac{3}{4}\right)$



2) $W = \{ y \in \mathbb{R} \mid y \geq -1 \}$

$= [-1 ; +\infty)$

$f_1(x) = e^x + e^{-2x}$

$f_1' = e^x - 2e^{-2x}$

$f_1'' = e^x + 4e^{-2x}$

$f_1' = 0 = e^x - 2e^{-2x}$

$e^x - \frac{2}{(e^x)^2} = 0$

$e^x = +\frac{2}{(e^x)^2}$

$(e^x)^3 = 2$

$\ln((e^x)^3) = \ln 2$

$3(\ln e^x) = \ln 2$

$x = \frac{\ln 2}{3}$

$f_1''\left(\frac{\ln 2}{3}\right) = e^{\frac{\ln 2}{3}} - 2e^{-2\frac{\ln 2}{3}}$

> 0

$\rightarrow$ Minimum

$f_2(x) = \frac{\ln x}{x}$

$f_2' = \frac{u'v - v'u}{v^2}$

$= \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2}$

$= \frac{1 - \ln x}{x^2}$

$u = 1 - \ln x$

$v = x^2$

$f_2'' = \frac{-x - 2x(1 - \ln x)}{x^4}$

$= \frac{-3x + 2x \ln x}{x^4}$

EP:

$0 = 1 - \ln x$

$x = e$

$f_2''(e) = \frac{-3e + 2e \cancel{\ln e}}{e^4}$

$= \frac{-e}{e^4} = -e^{-3}$

$\rightarrow$ Max

$f_3(x) = x^3 e^{-x}$

$f_3' = u'v + uv'$

$= 3x^2 e^{-x} - x^3 e^{-x}$

$= e^{-x}(-x^3 + 3x^2)$

$= e^{-x}(x^2)(-x + 3)$

$0 = (x^2)(-x + 3)$

$= -x^3 + 3x^2$

$x^3 = 3x^2$

$x = 3$

$f_3'(2) = e^{-2}(-8 + 12)$

$= e^{-2} \cdot 4 = \frac{4}{e^2} > 0$

$f_3'(4) = e^{-4}(16)(-1)$

$= \frac{-16}{e^4} < 0$

$\rightarrow$ VZW

$f_4(x) = x \cdot (\ln x)^3$

$u = x \qquad v = (\ln x)^3$

$u' = 1 \qquad v' = 3(\ln x)^2 \cdot \frac{1}{x}$

$f_4' = 3(\ln x)^2 + (\ln x)^3$

$= 3(\ln x)^2 + (\ln x)^2 (\ln x)$

$= (\ln x)^2 (3 + \ln x)$

$0 = (\ln x)^2 (3 + \ln x)$

$\Downarrow$

$x_1 = 1$

$0 = \ln x + 3$

$-3 = \ln x$

$e^{-3} = x$

VZW:

$f'(e^{-4}) = (\ln e^{-4})^3 + 3(\ln e^{-4})^2$

$= (-4)^3 + 3(-4)^2$

$= -64 + 48$

$= -16 \quad < 0$

$f'(e^{-2}) = (-2)^3 + 3(-2)^2$

$= -8 + 12 = +4$

> 0

$\rightarrow$ VZW

$f_t(x) = \frac{1}{2}(t \cdot x - \ln x) \qquad t > 0$

$f_t'(x) = \frac{1}{2}\left(t - \frac{1}{x}\right)$

$0 = \frac{1}{2}t - \frac{1}{2x}$

$\frac{1}{x} = t$

$x = \frac{1}{t}$

$f_t\left(\frac{1}{t}\right) = \frac{1}{2}\left(1 - \ln \frac{1}{t}\right)$

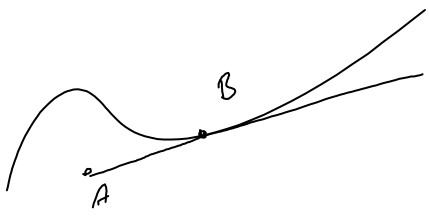
$= \frac{1}{2} - \frac{1}{2} \ln \frac{1}{t}$

$f_t''(x) = \frac{1}{2}\left(0 + \frac{1}{x^2}\right) = \frac{1}{2x^2}$

$\frac{1}{2}(1 + \ln t)$

$T\left(\frac{1}{t} ; \frac{1}{2} - \frac{1}{2} \ln \frac{1}{t}\right)$

$A(0 \mid 0{,}5)$



Taylorreihen und Anwendungen

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$f(x) = \underbrace{T_k(x)}_{\text{Taylorpolynom vom Grad } k} + \text{Rest}(x)$$

Rest(x) ↘ klein

$$f(x) \approx T_k(x) = \sum_{n=0}^{k} \frac{f^{(n)}(a)}{n!} (x-a)^n$$

fast

a: Entwicklungsstelle

Bsp.: <u>Weg-Zeit-Gesetz</u>

$$s(t) = b \cdot t^2 + v \cdot t + c$$

Punkt bewegt sich nach dem Weg-Zeit-Gesetz zur Zeit $\boxed{t=d}$

$$s(t) = s(d) + s'(d)(t-d) + \frac{1}{2} s''(d)(t-d)^2$$

$s(t)$ = Lage

$s'(t)$ = Geschw.

$s''(t)$ = Beschl.

$$f(x) = \frac{f^{(0)}(d)}{0!}(x-d)^0 + \frac{f'(d)}{1!}(x-d)^1 + \frac{f''(d)}{2!}(x-d)^2 + \underline{\underline{\text{höhere}}}$$

Taylorentwicklung

$$f(x) = x^4 - 5x^3 + 5x^2 + x + 2$$

Entwickle $f(x)$ in eine Taylorreihe um die Stelle $a = 2 = x_0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n = \frac{-7}{1}(x-2)^1 + \frac{-2}{2}(x-2)^2 + \frac{18}{6}(x-2)^3 + \frac{24}{24}(x-2)^4 + 0$$

$$f^{(0)}(2) = 16 - 5 \cdot 8 + 5 \cdot 4 + 2 + 2 = 0$$

$$f'(2) = 4x^3 - 15x^2 + 10x + 1 = 4 \cdot 8 - 15 \cdot 4 + 20 + 1 = -7$$

$$f''(2) = 12x^2 - 30x + 10 = 12 \cdot 4 - 30 \cdot 2 + 10 = -2$$

$$f'''(2) = 24x - 30 = 48 - 30 = 18$$

$$f^{(IV)}(2) = 24$$

$$f^{(V)}(2) = 0$$

$$= -7(x-2) - (x-2)^2 + 3(x-2)^3 + (x-2)^4$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$$(x-2)^4 = x^4 2^0 - 4x^3 \cdot 2^1 + 6x^2 \cdot 2^2 - 4x^1 \cdot 2^3 + 1 \cdot 2^4$$
$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

$$3(x-2)^3 = 3\left(x^3 2^0 - 3x^2 2^1 + 3x^1 2^2 - 1\, 2^3\right)$$
$$= 3\left(x^3 - 6x^2 + 12x - 8\right)$$
$$= 3x^3 - 18x^2 + 36x - 24$$
$$-(x-2)^2 = -x^2 + 4x - 4$$
$$-7(x-2) = -7x + 14$$

$$f(x) = x^4 - 5x^3 + 5x^2 + x + 2$$

Anwendung:

Einstein – relativistische Masse $m = \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

m_0 = Ruhemasse
v = Geschw.
c = Lichtgeschw.

$E = mc^2$

kinet. Energie

$$E_{kin} = mc^2 - m_0c^2$$
$$= \frac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \cdot c^2 - m_0c^2$$
$$= m_0c^2\left[\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right]$$

Taylor-entw.

$$= m_0c^2\left[\cancel{1} + \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{3}{8}\left(\frac{v}{c}\right)^4 + \cancel{\text{höhere}} \cancel{-1}\right]$$
$$= \frac{m_0c^2v^2}{2\,c^2} + \frac{3m_0c^2v^4}{8\,c^4} + 0$$
$$= \frac{m_0}{2}v^2 + \frac{3m_0}{8}\frac{v^4}{c^2} \approx E_{kin}$$

$x = \frac{v}{c}$

Taylorentw.

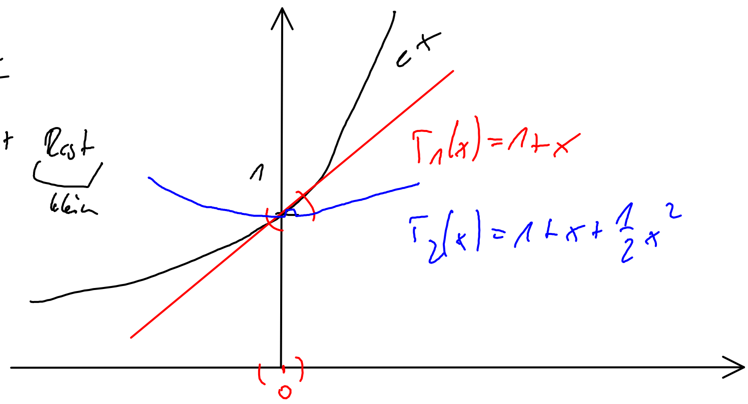
$$f(x) = \frac{1}{\sqrt{1-x^2}} = 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + \frac{5}{16}x^6 + \ldots + \text{höhere}$$

für $|x| < 1$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \ldots$$ im Punkt $a = 0$

Taylorreihe

$$f(x) = \underbrace{T_n(x)}_{\text{"leicht"}} + \underbrace{\text{Rest}}_{\text{klein}}$$



$f(x) = \left(1 - e^{-(x-2)}\right)^2 \approx$ Parabel

Berechne die Näherungsparabel in der Nähe des Minimums (2/0)

$$f'(x) = 2\left[e^{-(x-2)} - e^{-2(x-2)}\right]$$
$$f''(x) = 2\left[-e^{-(x-2)} + 2\cdot e^{-2(x-2)}\right]$$

$$0 = f'(x) = 2\left[e^{-(x-2)} - e^{-2(x-2)}\right]$$
$$e^{-2(x-2)} = e^{-(x-2)}$$
$$-2(x-2) = -(x-2)$$
$$-2x + 4 = -x + 2$$

$x = 2$ → in $f''(x)$ einsetzen → 0 → Minimum (2/0)

$a = 2 = x_0$

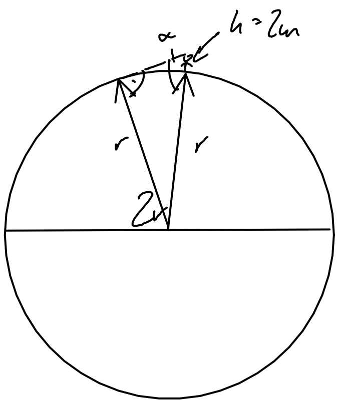
$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n \qquad f(x) = \left(1 - e^{-(x-2)}\right)^2$$

$$= \frac{f(2)}{0!}(x-2)^0 + \frac{f'(2)}{1!}(x-2)^1 + \frac{f''(2)}{2!}(x-2)^2 \qquad f(2) = 0$$

$$f'(2) = 0$$

$$= 0 + 0 + \frac{2}{2}(x-2)^2 = \underline{\underline{(x-2)^2}} \qquad f''(2) = 2$$

Erdradius 6000km und Kugel
Größe 2m
Wie weit kann man gucken?

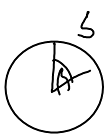


$\sin\alpha = \frac{r}{r + 2m}$

$\alpha = 89{,}95°$

$\cos\alpha = \frac{x}{r + 2m}$

$x \approx 4{,}899$ km



$$\frac{b}{\beta} = \frac{2\pi r}{360°} = \frac{\text{Umfang}}{\text{Vollwinkel}}$$

$\beta = 0{,}05°$

$b = 5{,}24$ km

Gute Nacht

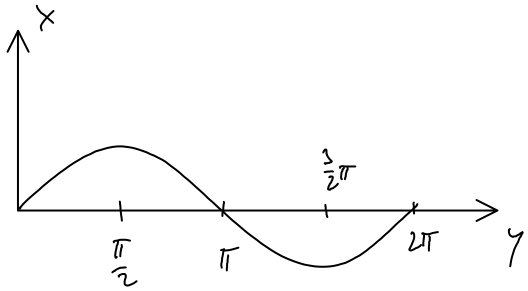
~~Hallo~~ :)

Hallo Dani

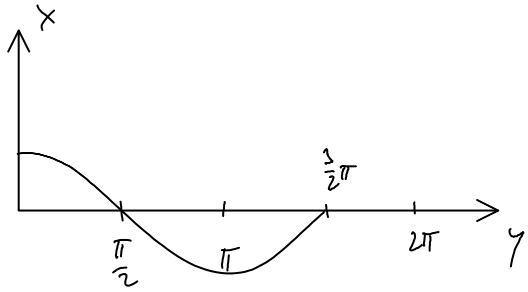
V. Ungel

Hallo

sin x – cos x Kurvendiskussion



$f(x) = \sin x$
$f'(x) = \cos x$
$f''(x) = -\sin x$
$f'''(x) = -\cos x$



$f(x) = \cos x$
$f'(x) = -\sin x$
$f''(x) = -\cos x$
$f'''(x) = \sin x$

$h(x) = \sin x \cdot \cos x$ $\qquad h'(x) = uv' + u'v = \sin x(-\sin x) + \cos^2 x = -\sin^2 x + \cos^2 x$

$k(x) = \sin 3x$ $\qquad k'(x) = 3\cos 3x$

$f(x) = \cos^2 x - \frac{1}{2}\cos x - 1$

NSt, Extremstellen

$0 = \cos^2 x - \frac{1}{2}\cos x - 1$ $\qquad \cos x = z$

$= z^2 - \frac{1}{2} z - 1$

$z_{1,2} = +\frac{1}{4} \pm \sqrt{\frac{(-1/2)^2}{4} + 1}$

$= \frac{1}{4} \pm \sqrt{\frac{17}{16}} = \frac{1}{4} \pm \frac{\sqrt{17}}{4}$

$z_1 = \frac{1+\sqrt{17}}{4}$ $\qquad z_2 = \frac{1-\sqrt{17}}{4}$

$\downarrow$

$z_1 = \cos x = \frac{1+\sqrt{17}}{4} > 1 \rightarrow$ kein Lsg.

$z_2 = \cos x = \frac{1-\sqrt{17}}{4} < 0 \rightarrow$ Lösung ex.

$x_1 = 141{,}33°$

$x_2 = 219°$ $\qquad b = \frac{2\pi \cdot \alpha}{360°}$ $\qquad b_1 = 2{,}47$

$b_2 = 3{,}82$

$f(x) = \cos^2 x - \frac{1}{2}\cos x - 1$

$f'(x) = 2\cos x(-\sin x) + \frac{1}{2}\sin x$

$= -2\cos x \sin x + \frac{1}{2}\sin x$

$= \sin x \left(-2\cos x + \frac{1}{2}\right)$

$0 = \sin x \left(-2\cos x + \frac{1}{2}\right) \longrightarrow \sin x \neq 0$

$\frac{1}{2} = 2\cos x$

$\cos x = \frac{1}{4}$

$x_1 = 1{,}32 \qquad x_2 = 4{,}97$

$x_3 = \pi \approx 3{,}14 \qquad x_4 = 0$

$f''(x) = 2\sin^2 x - 2\cos^2 + \frac{1}{2}\cos x \qquad u = \sin x \qquad v = -2\cos x + \frac{1}{2}$

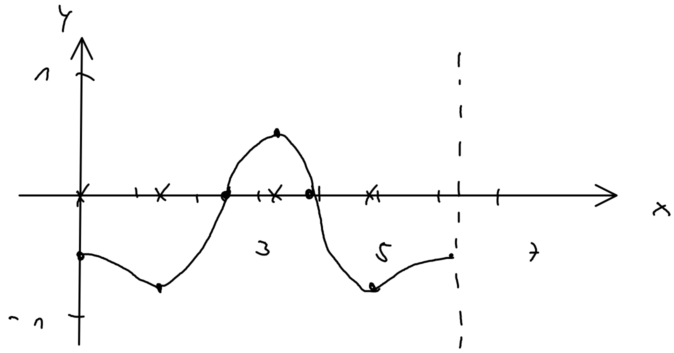
$= -4\cos^2 x + \frac{1}{2}\cos x + 2 \qquad u' = \cos x \qquad v' = 2\sin x$

$f''(x_1) = 1{,}88 > 0$ Minimum $\qquad T = (1{,}32 / -0{,}76)$

$f''(x_2) = 1{,}87 > 0$ Min $\qquad T = (4{,}97 / -0{,}77)$

$f''(x_4) = -1{,}5 = -\frac{3}{2} < 0$ Max $\qquad H = (0 / -0{,}5)$

$f''(x_3) = -\frac{5}{2} < 0$ Max $\qquad H = (\pi / 0{,}5)$



$f(x) = 3\sin x + 4\cos x$

$f'(x) = 3\cos x + (-4)\sin x$

$f''(x) = -3\sin x - 4\cos x$

$f(x) = 2\cos x + 2\sin x \cos x$

NSt:

$0 = 2\cos x + 2\sin x \cos x \qquad = \cos x\,(2 + 2\sin x)$

$0 = \cos x$

$x_1 = \frac{\pi}{2}$

$x_2 = \frac{3}{2}\pi$

$0 = 2 + 2\sin x$

$x_3 = \frac{3}{2}\pi$

$f'(x) = -2\sin x + 2\left[\sin x(-\sin x) + \cos x \cos x\right]$

$= -2\sin x + 2\left[1 - 2\sin^2 x\right]$

$= -2\sin x + 2 - 4\sin^2 x$

$f''(x) = -2\cos x - 8\sin x \cos x$

$f'(x) = 0$

$0 = -2\sin x + 2 - 4\sin^2 x \qquad \sin x = z$

$= -4\sin^2 x - 2\sin x + 2$

$= -4z^2 - 2z + 2$

$= z^2 + \frac{1}{2}z - \frac{1}{2}$

$z_{1,2} = \frac{1}{4} \pm \sqrt{\frac{(\frac{1}{2})^2}{4} + \frac{1}{2}}$

$z_1 = -\frac{1}{4} + \sqrt{\frac{9}{16}} = -\frac{1}{4} - \frac{3}{4} = -1$

$z_2 = -\frac{1}{4} - \sqrt{\frac{9}{16}} = -\frac{1}{2}$

$z_1 = \sin x \qquad x_1 = \frac{3}{2}\pi$

$z_2 = \sin x \qquad x_2 = \frac{1}{6}\pi$

$x_3 = \frac{5}{6}\pi$

$f''(x_1) = 0$

$f''(x_2) = \boxed{3} > 0$

04.11.2009

$f(x) = \frac{1}{3}x\sqrt{16-x^2} = \frac{1}{3}x \quad (16-x^2)^{\frac{1}{2}}$

$u \qquad v'$

$= \frac{1}{3}\left(\sqrt{16-x^2}\right) + (-2x)\cdot\frac{1}{2\sqrt{16-x^2}}\cdot\left(\frac{1}{3}x\right)$

$= \frac{\frac{1}{3}(16-x^2)-\frac{1}{3}x^2}{\sqrt{16-x^2}} = \frac{\frac{16}{3}-\frac{2}{3}x^2}{\sqrt{16-x^2}}$

11.11.2009

LGS

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$x+y+z=7$
$2x-2y+z=6$
$-3x+y-z=+9$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 2 & -2 & 1 & 6 \\ -3 & 1 & -1 & 9 \end{array}\right)$$

$I\cdot 2$ −

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -4 & -1 & -8 \\ -3 & 1 & -1 & 9 \end{array}\right)$$

$I\cdot 3$ +

$$\left(\begin{array}{ccc|c} 3 & 3 & 3 & 21 \\ 0 & -4 & -1 & -8 \\ 0 & 4 & 2 & 36 \end{array}\right)$$

+

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -4 & -1 & -8 \\ 0 & 0 & 1 & 28 \end{array}\right)$$

$x+y=1$
$y+z=1$
$3x+2y+z=0$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & -3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & -2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$x=-1$
$x+2-1=2$
$y-1=1 \Rightarrow y=2$
$z=-1$

Determinante

Matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{21}a_{12}$

$= ad - bc = \det|A| = \det\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\det\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = 0$

Cramersche Regel

$\det\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix} = 1 + 3 - 2 = 2$

$A \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \mathrel{\hat=} A \cdot x = b$

$x_1 = \frac{1}{\det(A)} \cdot \det\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$

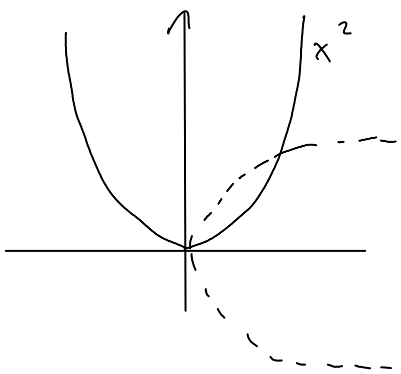
$= \frac{1}{2} \cdot -2 = \underline{\underline{-1}}$

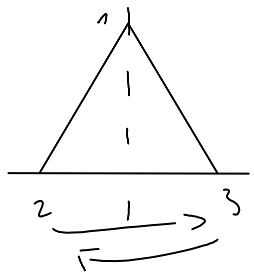
$x_2 = \frac{1}{\det(A)} \cdot \det\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix} = \frac{1}{2} \cdot (1+3) = \underline{\underline{2}}$

$x_3 = \frac{1}{\det(A)} \cdot \det\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 3 & 2 & 0 \end{pmatrix} = \frac{1}{2} \cdot (3 - 3 - 2) = \underline{\underline{-1}}$

Allgemeine Umkehrabbildung

$f(x) = x^2$





$2 \to 3$

$s_p \to s_p = \underline{\underline{id}}$

$s_p \circ s_p = id$

$\underset{(2)}{f} \circ \underset{(1)}{g} = id$

$= f(g(x))$

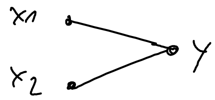
nicht inj. $\leftarrow$ $f: \mathbb{R} \to \mathbb{R}$

$x \to f(x) = x^2$ nicht umkehrbar

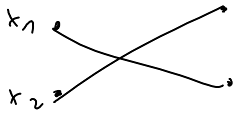
injektiv $\leftarrow$ $\tilde{f}: \mathbb{R}_{\geq 0} \to \mathbb{R}$ ist umkehrbar

Def.: $f: V \to W$ heißt umkehrbar, wenn f bijektiv ist
f ist injektiv und surjektiv

inj.: $f(-2) = 4 = f(2)$
$(-2)^2 = 4 = (2)^2$



$f: V \to W$ heißt injektiv, wenn gilt
$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$



oder aus $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

$A \Rightarrow B \iff \neg B \Rightarrow \neg A$
$\bar{B} \Rightarrow \bar{A}$

$g: \mathbb{N} \to \mathbb{N}$
$x \to x^2$
inj

$g: \mathbb{Z} \to \mathbb{N}$
$x \to x^2$
nicht inj

alle Abb.:
$\{1, 2\} \to \{1, 2, 3\}$

f_1 $1 \to 1$
f_1 $2 \to 1$
nicht inj.

$1 \to 2$ $2 \to 2$	$1 \to 2$ $2 \to 1$	$1 \to 1$ $2 \to 3$	$1 \to 2$ $2 \to 3$	$1 \to 3$ $2 \to 2$
$1 \to 3$ $2 \to 3$	$1 \to 1$ $2 \to 2$	$1 \to 2$ $2 \to 3$	$1 \to 3$ $2 \to 1$	

THOMAS
SPAST

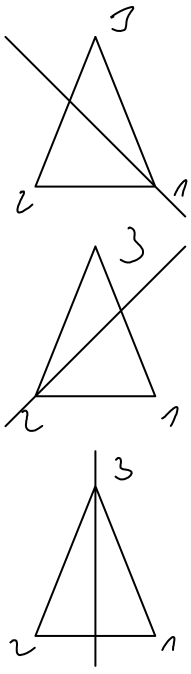
$f: \{1, 2, 3\} \to \{1, 2\}$

f_1 | $1 \to 1$, $2 \to 1$, $3 \to 2$ — surjektiv

f_2 | $1 \to 2$, $2 \to 2$, $3 \to 2$ — injektiv

11.11.2009

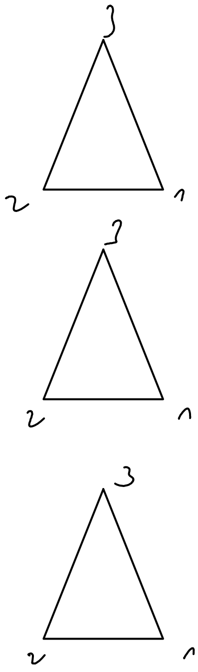
123	σ_1
132	σ_2
213	σ_3
231	σ_4
312	σ_5
321	σ_6



$\begin{pmatrix} 1 2 3 \\ 1 3 2 \end{pmatrix} = \sigma_4$

$\begin{pmatrix} 2 1 3 \\ 2 3 1 \end{pmatrix}$

$\begin{pmatrix} 3 1 2 \\ 3 2 1 \end{pmatrix}$



$g: \{1,2,3\} \to \{1,2,3\}$

$1 \to 2$
$2 \to 2$
$3 \to 1$

nicht surj.
nicht inj.

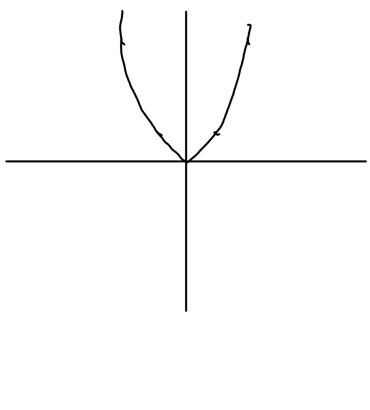
* $f: V \to W$ heißt surjektiv, wenn gilt $\forall\, y \in W \ \exists\, \tilde{x} \in V$
$f(\tilde{x}) = y$

18.11.2009

Wdh. Wann umkehrbar?

$f_1: \mathbb{R}_{>0} \to \mathbb{R}_{>0}$
$x \to x^2$

$f_2: \mathbb{R} \to \mathbb{R}$
$x \to x^2$



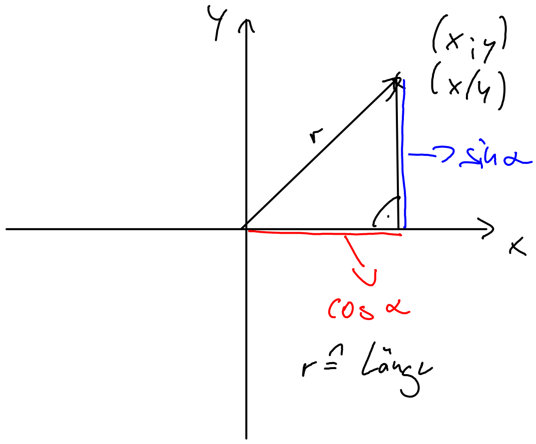
$f_3: \mathbb{R}_{<0} \to \mathbb{R}$

f_1 umkehrbar

$f: \mathbb{N} \to \mathbb{N}$ bij.

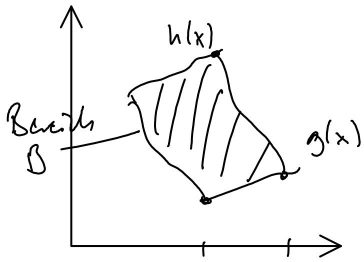
$f: \mathbb{N}_0 \to \mathbb{N}$ bij., da $n \to n+1$

Polarkoordinaten



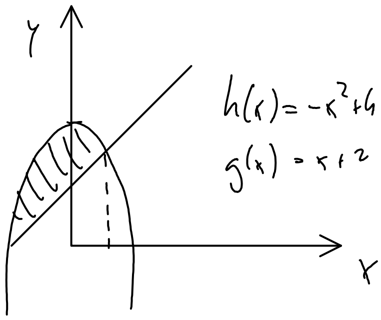
$x ≙ r \cdot \cos\alpha$

$y ≙ r \cdot \sin\alpha$



Um Flächen zu berechnen wird die Funktion 1 integriert.

$$\iint_B 1 \, dy \, dx =$$



$h(x) = -x^2+4$

$g(x) = x+2$

$$A = \int_{-2}^{1}\int_{g(x)}^{h(x)} 1 \, dy \, dx = \int_{-2}^{1}\int_{x+2}^{-x+2} 1 \, dy \, dx$$

Schnitt : $x+2 = -x^2+4$

$x^2+x-2 = 0$

$$\int_{x+2}^{-x^2+4} 1 \, dy = -x^2+4-x-2 = \underline{-x^2-x+2}$$

$$A = \int_{-2}^{1} -x^2-x+2 \, dx = \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x\right]_{-2}^{1} = \frac{7}{6} - \left(-\frac{10}{3}\right) = 4{,}5$$

Schwerpunkt $S(x_s, y_s)$:

$$x_s = \frac{1}{A}\iint_B x \, dy \, dx = \int_{-2}^{1}\left[\int_{x+2}^{-x^2+4} x \, dy\right] dx$$

$$y_s = \frac{1}{A}\iint_B y \, dy \, dx$$

⇝ $x_S: \int_{-2}^{1}\left[x\int_{x+2}^{-x^2+4} 1\,dy\right]dx = \int_{-2}^{1}\left[x(-x^2+4-x-2)\right]dx = \int_{-2}^{1}\left[-x^3+4x-x^2-2x\right]dx$

$$= \left[-\frac{1}{4}x^4+2x^2-\frac{1}{3}x^3-x^2\right]_{-2}^{1} = \left[-\frac{1}{4}x^4-\frac{1}{3}x^3+x^2\right]_{-2}^{1}$$

$$= \frac{5}{12}-\left(\frac{8}{3}\right) = \underline{\underline{-\frac{9}{4}}}$$

⇝ $x_S = \frac{1}{A}\cdot -\frac{9}{4}$

$= \frac{1}{4{,}5}\cdot -\frac{9}{4} = \underline{\underline{-0{,}5}}$

$y_S = \frac{1}{A}\iint_B y\,dy\,dx = \frac{1}{A}\int_{-2}^{1}\left[\int_{x+2}^{-x^2+4} y\,dy\right]dx$

⇝ $\int_{-2}^{1}\left(\left[\frac{1}{2}y^2\right]_{x+2}^{-x^2+4}\right)dx = \int_{-2}^{1}\left[\left(\frac{1}{2}\cdot(-x^2+4)^2-\frac{1}{2}(x+2)^2\right)\right]dx$

$= \int_{-2}^{1}\left[\frac{1}{2}\left(x^4-8x^2+16-(x^2+4x+4)\right)\right]dx = \int_{-2}^{1}\left[\frac{1}{2}\left(x^4-9x^2-4x+12\right)\right]dx$

$= \left[\frac{1}{2}\left(\frac{1}{5}x^5-3x^3-2x^2+12x\right)\right]_{-2}^{1}$ $\qquad F(1)=\frac{18}{5}$

$\qquad F(-2) = -\frac{36}{5}$

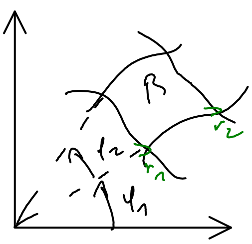
$= \frac{18}{5}-\left(-\frac{36}{5}\right) = \frac{54}{5} = 10{,}8$

⇝ $y_S = \frac{1}{A}\cdot 10{,}8$

$y_S = \frac{1}{4{,}5}\cdot 10{,}8$

$\underline{\underline{y_S = 2{,}4}}$ $\qquad$ ⇝ $S(-0{,}5/2{,}4)$

Polarkoordinaten



r geht von r_1 bis r_2

$\iint 1$

$\iint_B 1\,\underbrace{dy\,dx}_{r\,dr\,d\varphi}$ $\qquad f(x,y) \Rightarrow f(r\cos\varphi, r\sin\varphi)$

$x = r\cos\varphi \qquad \to dr\,d\varphi$

$y = r\sin\varphi$

$$\det\begin{pmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{pmatrix} \Rightarrow \text{J-Matrix} \begin{pmatrix} \frac{dx}{dr} & \frac{dx}{d\varphi} \\ \frac{dy}{dr} & \frac{dy}{d\varphi} \end{pmatrix}$$

$$\Downarrow$$

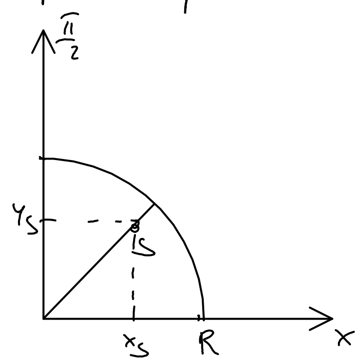
$$= r\cos^2\alpha + r\sin^2\alpha$$
$$= r(\cos^2\alpha + \sin^2\alpha)$$
$$= \underline{\underline{r\cdot 1}} \quad \text{(Korrekturfkt.)}$$

$$\frac{dx}{dr}\cdot\frac{dy}{d\varphi} - \frac{dy}{dr}\,\frac{dx}{d\varphi}$$

det() ⇒ Volumenbegriff von Vektoren

det Zahl als Ergebnis ⇒ Volumen

Bsp. Schwerpunkt Viertelkreis



$$x_S = \frac{1}{A}\iint_{\frac{\pi}{2}\,R} x\,dy\,dx$$

Korrektur-r

$$= \frac{1}{A}\int_0\int_0 (r\cos\varphi)\,r\,dr\,d\varphi$$

$$A_{\frac{1}{4}Kr} = \frac{R^2\pi}{4}$$

$$x_S = \frac{1}{A}\int_0^{\frac{\pi}{2}} \underbrace{\left[\int_0^R (r\cos\varphi)\,r\,dr\right]}\,d\varphi$$

$$= \frac{1}{A}\int_0^{\frac{\pi}{2}}\left[\int_0^R r^2\cos\varphi\,dr\right]d\varphi = \frac{1}{A}\int_0^{\frac{\pi}{2}}\cos\varphi\left[\frac{1}{3}r^3\right]_0^R d\varphi$$

$$= \frac{1}{A}\int_0^{\frac{\pi}{2}}\cos\varphi\left(\frac{1}{3}R^3\right)d\varphi = \frac{1}{A}\cdot\frac{1}{3}R^3\int_0^{\frac{\pi}{2}}\cos\varphi\,d\varphi$$

$$= \frac{1}{A}\cdot\frac{1}{3}R^3\left[\sin\varphi\right]_0^{\frac{\pi}{2}} = \frac{1}{A}\cdot\frac{1}{3}R^3(1-0) = \frac{1}{\frac{R^2\pi}{4}}\cdot\frac{1}{3}R^3\cdot 1$$

$$= \frac{4}{R^2\pi}\cdot\frac{1}{3}R^3 = \underline{\underline{\frac{4}{3\pi}R}}$$

$\Rightarrow x_S = \frac{4}{3}\pi R$, da symmetrisch $\Rightarrow y_S = \frac{4}{3}\pi R$

$$\Rightarrow S\left(\frac{4}{3\pi}R \,/\, \frac{4}{3\pi}R\right)$$

Substitution

$$\int_1^2 f(g(x))\cdot g'(x)\,dx = \int_{g(1)}^{g(2)} f(z)\,dz$$

$$\int_0^{\pi} \cos\left(\frac{1}{3}x^2\right)\cdot \frac{1}{3}2x\,dx = \int_{\frac{1}{3}0^2}^{\frac{1}{3}\pi^2} \cos z\,dz \qquad \left(\frac{1}{3}x^2\right)' = \frac{1}{3}2x$$

$$\underline{z = \frac{1}{3}x^2}$$

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hier taucht keine Ableitung auf ⇒ Substituieren

$$\int \frac{e^x-1}{e^x+1}\,dx \qquad \text{Subst: } t = e^x+1$$

$$t-1 = e^x$$

$$\ln(t-1) = x \qquad e^x - 1 = \underbrace{(e^x+1)}_{t} - 2$$

$$\frac{dx}{dt} = \frac{1}{t-1} \qquad e^x-1 = t-2$$

$$\int \frac{t-2}{t}\,\frac{1}{t-1}\cdot dt \qquad dx = \frac{1}{t-1}\,dt$$

$$\int\left(\frac{t}{t(t-1)} - \frac{2}{t(t-1)}\right)dt = \int \frac{1}{t-1}\,dt - 2\int\frac{1}{t(t-1)}\,dt$$

$$\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \qquad |\cdot t$$

$$\frac{1}{t-1} = \frac{A}{t}\cdot t + \frac{B}{t-1}\cdot t \qquad \text{Setze } t=0 \text{ um } B \text{ rauszuschmeißen}$$

$$\frac{1}{-1} = A$$

$$\underline{-1 = A}$$

$$\frac{1}{t} = \frac{A}{t}(t-1) + B \qquad t = +1 \text{ um } A \text{ rauszuschmeißen}$$

$$\frac{1}{1} = 0 + B$$

$$\underline{1 = B}$$

$$\Rightarrow \int \frac{1}{t-1}\,dt - 2\left[\int \frac{-1}{t}\,dt + \int\left(\frac{1}{t-1}\right)dt\right]$$

$$\ln(t-1) - 2\left[-\ln t + \ln(t-1)\right] = -\ln(t-1) + 2\ln t$$

$$= -\ln(e^x) + 2\ln(e^x+1)$$

$$\underline{\underline{F = -x + 2\ln(e^x+1)}}$$

$$F' = -1 + 2\,\frac{1}{e^x+1}\cdot e^x = -1 + \frac{2e^x}{e^x+1} = \frac{(e^x+1)-1}{e^x+1} + \frac{2e^x}{e^x+1}$$

$$= \frac{-e^x-1+2e^x}{e^x+1} = \underline{\underline{\frac{1e^x-1}{e^x+1}}} = f(x)$$